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1. Learning outcomes

To provide a practical and theoretical introduction to the stereographic projection to enable students to use it in morphological crystallography

2. Crystal Projections

A Crystal projection is a quantitative method of representing a three dimensional crystal on a two dimensional planar surface. Different projections are used for different purposes but each has its own set of rules so that the projection bears a known and reproducible relationship to the crystal. It involves a series of steps that develop these projections to show how they can be used to represent the symmetry inherent in crystals. These then allows for detailed graphical images of points groups assigned to each crystal class of the six crystal systems.

3. Principles

In the study of crystallography it is often useful to be able to represent crystal planes and crystal directions on a diagram in two dimensions so that angular relationships and the symmetrical arrangements of crystal faces can be discussed upon a flat piece of paper, and if required, *measured*. The most useful type of diagram will be one in which the angular relationships in three dimensions in the crystal, are faithfully reproduced in a plane in some form of projectional geometry.

Mathematically, a projection from three dimensions to two dimensions in which angular relationships are faithfully reproduced is known as a *conformal projection*. The conformal projection used in crystallography is the *stereographic projection.*

The stereographic projection is a very ancient geometrical technique; it originated in the second century A.D. in the work of the Alexandrian astronomer Claudius Ptolemy who used it as a means of representing the stars on the heavenly sphere. The original Greek manuscript is lost, but the work comes down to us in a sixteenth-century Latin translation from an Arabic commentary entitled The Planispherium.

The stereographic projection was first applied to crystallography in the work of F. E. Neumann* and was further developed by W. H. Miller.

In order to visualize how a stereographic projection is used in crystallography, let us imagine a crystal to be positioned with its center at the center of a sphere, which we call the *sphere of projection* (**Fig. 1a**), and draw normals to crystal planes through the center of the sphere to intersect the surface of the sphere, say at P. P is called the *pole* of the plane of which OP is the normal. A direction is similarly represented by a point on the surface of the sphere, defined as the point where the line parallel to the given direction, passing through the center of the sphere, strikes the surface of the sphere.

A crystal plane can also be represented by drawing the parallel plane through the center of the sphere and extending it until it strikes the sphere (**Fig. 1a**). Since the plane passes through the center of the sphere, it is a *diametral plane*, and the line of intersection of the sphere with such a plane is called a *great circle*. A great circle is a circle on the surface of a sphere with radius equal to the radius of the sphere.

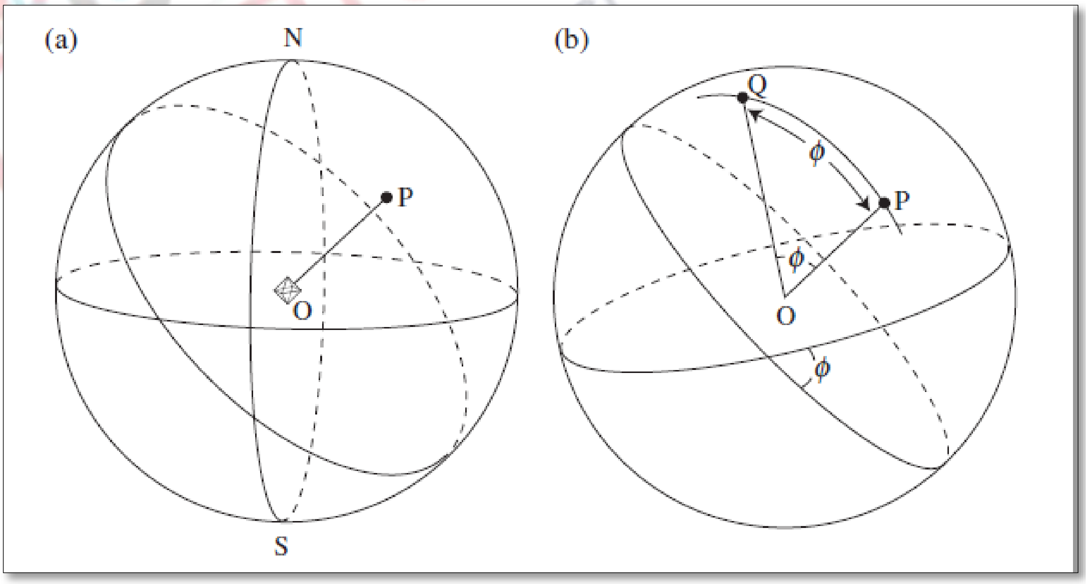

Fig. 1 (a) Sphere of projection. (b) The angle between two planes is equal to the angle φ between the two poles.

At this stage, we have represented directions in the crystal – that is, normals to lattice planes or lattice directions – by points (poles) on the surface of the sphere.

This is called as *spherical projection* of the crystal. The angle between two planes of which the normals are OP and OQ (**Fig. 1b**) is equal to the angle between these normals, which is the angle subtended at the center of the sphere of projection by the arc of the great circle drawn through the poles P and Q. In order to make a drawing in two dimensions in which angular relationships are preserved, we now project the poles on to a suitable two-dimensional plane such as a piece of paper.

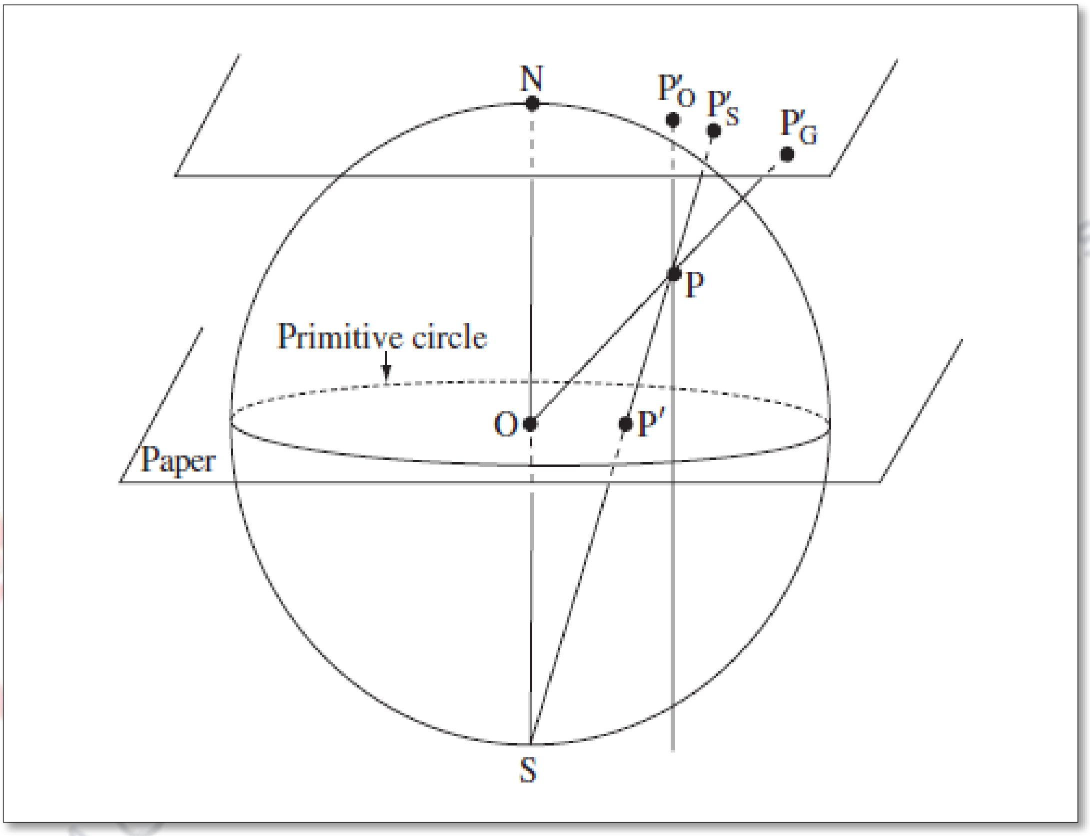

Fig. 2 Schematic illustrating various types **f** projections for plotting a face pole.

The spherical projection is like a terrestrial globe. Let us define north and south poles, N and S in Fig. 2, by analogy with the north and south poles of a globe. The equatorial plane passes through the center of the sphere normal to the line NS and cuts the sphere in a great circle called the equator.

There are various ways of projecting points on the sphere on to a two-dimensional plane as shown in Fig. 2. In the *orthographic* projection, a pole P is projected from a point at infinity on to a plane parallel to the equatorial plane to form P'_O on a plane parallel to the equatorial plane passing through N. In the *gnomonic* projection, the point of projection is the center of the sphere, giving the projected pole at P'_G on a plane parallel to the equatorial plane passing through N. Both these projections have their uses in crystallography – the orthographic projection is useful for visualizing crystal shapes and the gnomonic projection is relevant for labelling electron back-scattered electron diffraction patterns in scanning electron microscopes. However, neither of these projections is conformal, and so angles are distorted in these projections.

In the *stereographic* projection the pole P is projected from a point on the surface of the sphere, say S, called the pole of projection, on to a plane normal to OS. This plane can pass through any point on NS. If it passes through N, the point P projects to P'_S. The most convenient plane for our purpose is the equatorial plane normal to SO. If we project the point P from S on to this plane, we define the point P′ so produced as the *stereographic projection* of P. As rule the plane of projection as the equatorial plane. The line of intersection of the plane of projection with the sphere of projection is a great circle called the primitive circle.

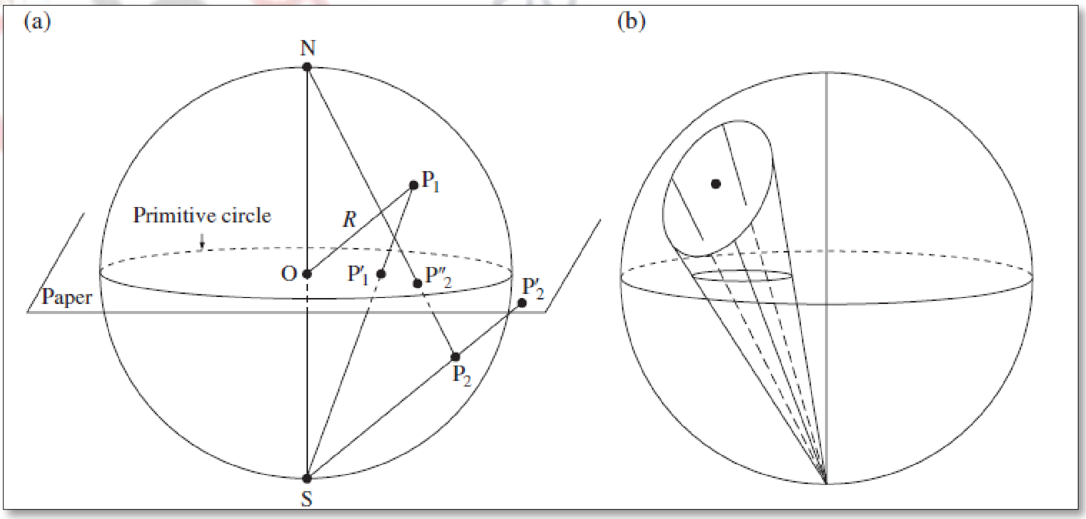

Fig. 3 (a) Schematic illustrating concept of true projection. (b) A small circle projects as a circle.

The method of projection can be visualized from **Fig. 3(a).** As can be seen on **Fig. 3(a)** a pole P_1 in the northern hemisphere projects to P'_1, inside the primitive, and is marked with a dot on the paper. All poles in the northern hemisphere project inside the primitive. Poles in the southern hemisphere, say P_2, give a projection P'_2 outside the primitive. The point P'_2 is the *true* projection of P_2. It is often inconvenient to work with projected poles outside the primitive, and to avoid this a pole P_2, in the southern hemisphere, may be projected from the north pole N (diametrically opposite S) to give the projected pole at P''_2. The projected pole P''_2 is then distinguished from the *true projection* of P_2 (at P'_2) by marking the point P''_2 with a ring instead of with a dot. In addition to being angle true, the stereographic projection has a second very useful property: all circles (great or small) on the surface of the sphere of projection project as circles. This is illustrated for a small circle in **Fig. 3b**.

4. Stereographic net

The measurement and plotting of angles on the stereographic projection is made easier by using a template of the projected coordinate system, a stereographic net. This type of net is also known as the Wulff net, after G. V. Wulff, Russian Crystallographer (1863-1925) (**Fig. 4**).

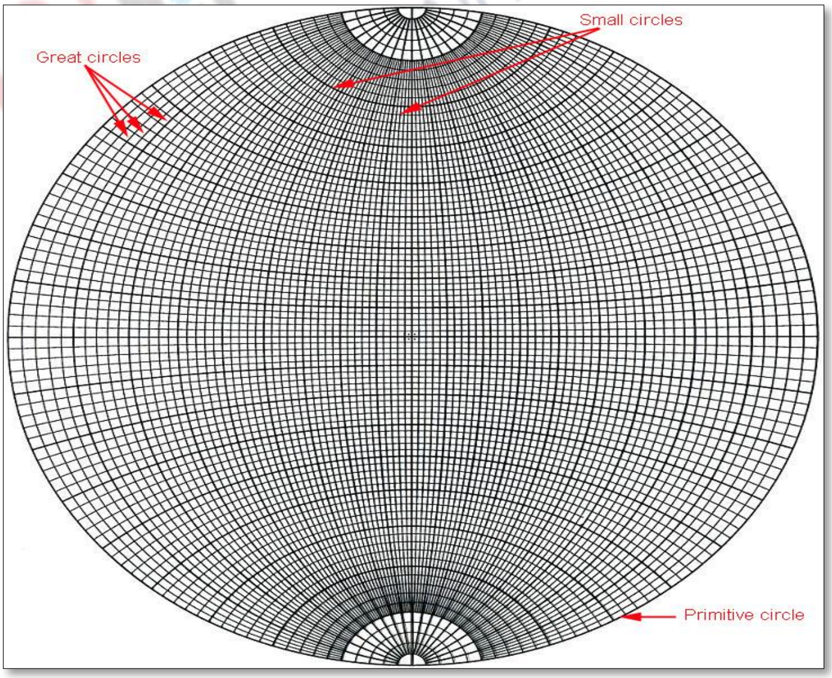

Fig. 4 Wulff Net. Marked on it are great circle, small circle and primitive circle.

In the stereonet, the **N** and **S** poles would plot directly above and below the center of the stereonet. **Fig. 4** shows several different components as defined below.

a) **The Primitive Circle** is the circle that surrounds the stereonet.

b) **Great Circles** are the curved lines that connect the points labeled N and S on the stereonet. The **E-W** and **N-S** axes, as well as the Primitive Circle are also great circles. Angular relationships between points can only be measured on Great Circles.

c) **Small Circles** are the highly curved lines that curve upward and downward on the stereonet.

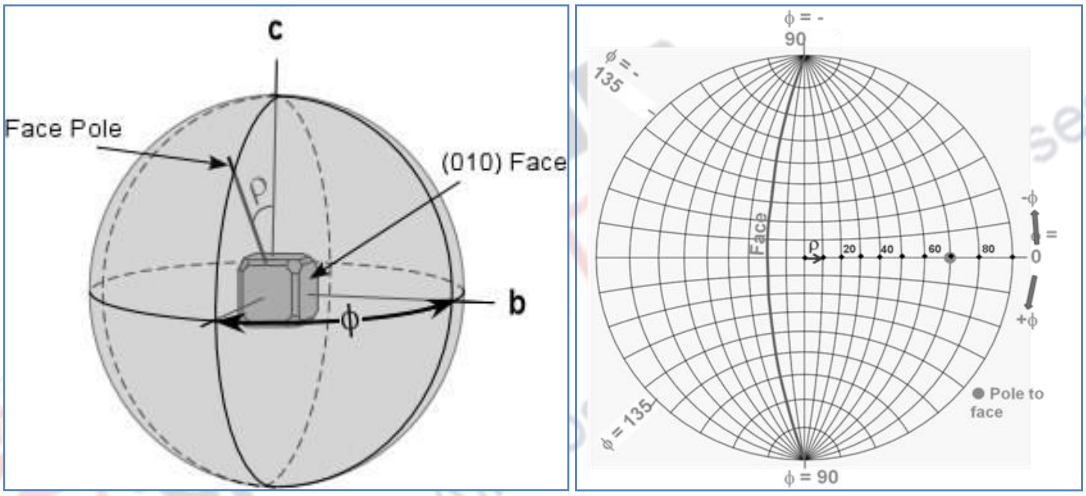

Fig. 5 (a) Imagine that we have a crystal inside of a sphere. From each crystal face we draw a line perpendicular to the face (poles to the face). We define this face (010) as having a φ angle of 0º. For any other face, the φ angle will be measured from the b axis in a clockwise sense in the plane of the equator. We define the ρ angle, as the angle between the c axis and the pole to the crystal face, measured downward from the North pole of the sphere. In the figure we can observe that a crystal face has a ρ angle measured in the vertical plane containing the axis of the sphere and the face pole, and a φ angle measured in the horizontal equatorial plane. Note that the (010) face has a ρ angle = 90º. (b) In order to make plotting of the stereographic projection easier, a device called a stereographic net or stereonet is used.

The following rules are applied for plotting crystal faces on the stereonet:

- ✓ All crystal faces are plotted as poles.

- ✓ The b crystallographic axis is taken as the starting point. Such an axis will be plot at φ = 0º and ρ = 90º.
- ✓ Positive φ angles will be measured clockwise on the stereonet, and negative φ angles will be measured counter-clockwise on the stereonet.
- ✓ Crystal faces that are on the top of the crystal (ρ <90) will be plotted as open circles, and crystal faces on the bottom of the crystal (ρ > 90) will be plotted as "+" signs.
- ✓ Place a sheet of tracing paper on the stereonet and trace the outermost great circle. Make a reference mark on the right side of the circle (East).
- ✓ To plot a face, first measure the φ angle along the outermost great circle, and make a mark on your tracing paper. Next, rotate the tracing paper so that the mark lies at the end of the E-W axis of the stereonet.
- ✓ Measure the ρ angle out from the center of the stereonet along the E-W axis of the stereonet. Note that angles can only be measured along great circles. These include the primitive circle, and the E-W and N-S axis of the stereonet.
- ✓ Any two faces on the same great circle are in the same zone. Zones can be shown as lines running through the great circle containing faces in that zone. The zone axis can be found by setting two faces in the zone on the same great circle, and counting 90 away from the intersection of the great circle along the E-W axis.

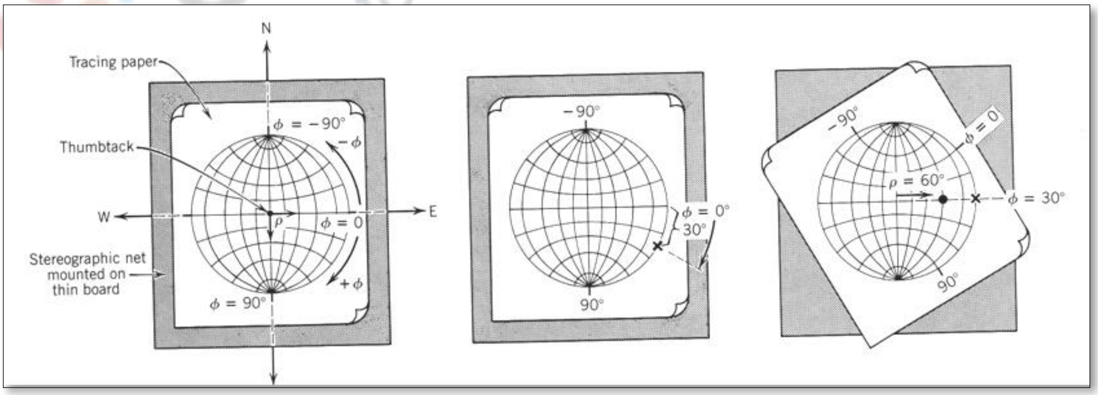

Fig. 6 Illustration of the use of stereographic net. Suppose we measured ρ = 60º and φ = 30º for a face with goniometer. Plot the pole to this face on the stereonet. Procedure: Line up the N of the tracing paper with the N of the net. From E, count 30 clockwise; put an x (or a tick mark). Bring x to the E, and then count 60 from the center toward E, along the E-W line. Mark the point with ●

5. Summary

Stereographic projection provides a means of representing different planes and directions of a crystal in two dimensions. It allows measurement of angles between planes and directions. Stereographic projection is an angle true projection. The angular relation between different directions is maintained but not the linear distances. In stereographic projection, the crystal is imagined to be at the center of a sphere (the stereographic sphere); the normals to the crystal faces are imagined to radiate out from the center and to intersect the sphere in an array of points. Each point on the sphere therefore represents a crystal face or plane (and is labelled with the appropriate Miller index). The stereographic projection is used to represent the angles between the faces of a crystal and the symmetry relationships between them. Imagine that the crystal is centered within a sphere, the normal to the crystal faces will give a consistent set of points uninfluenced by the relative sizes of the faces. The symmetry of the arrangement of these points on the sphere reveals the symmetry of the crystal. In "Spherical Projection" a crystal is positioned with its center at the center of a sphere. This sphere is called as "reference sphere" or "sphere of projection". Crystal planes and directions can be represented on the surface of this sphere.

Frequently Asked Questions-

Q1. Define Crystal projection?

Ans. A crystal projection is a quantitative method of representing a three dimensional crystal on a two dimensional planar surface. Crystal projections have some definite rules so that the projection bears a known and reproducible relationship to the crystal.

Q2. Define a great circle?

Ans. If any plane is passed through a sphere, it will intersect the surface of the sphere in a circle. The circles of maximum diameter are those formed by plane passing through the center and having a diameter equal to the diameter of the sphere. These are called great circles in crystal projections.

Q3. How do we evaluate which crystals faces belong to a zone in crystal projections?

Ans. To determine which crystal faces belong to a zone, we simply evaluate which face poles lie along the same great circle. All of the faces that belong to a zone lie along the same great circle of projection.

Multiple Choice Questions-

1. For stereographic projections we use

(a) Wulff Net
(b) Schmidt Net
(c) Michael levy chart

Ans: a

2. The crystal longitude (azimuth, φ) of a face pole in a spherical projection is measured in degrees up to

(a) 180°
(b) 360°
(c) 90°

Ans: a

3. Circle of maximum diameter in Wulff net is called

(a) Great circle
(b) Small circle
(c) Longitude

Ans: a

4. For measuring crystal angles we use

(a) Universal stage
(b) Goniometers
(c) Compass

Ans: b

5. Polar angle "ρ" is measured in degrees from

(a) North pole
(b) South pole
(c) Equator

Ans: a

Suggested Readings:

1. Klien, Cornelis and Dutrow, Barbara, (2008). Manual of Mineral Sciences (Manual of Mineralogy), 23rd Edn. John Wiley & Sons, New York. ISBN: 0471721573, 978-0471721574.
2. Nesse, William D. (2016). Introduction to Mineralogy, 3rd Edn. Oxford University press. ISBN: 9780190618353.
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4. Bloss, F. Donald (1971). Crystallography and Crystal Chemistry. Holt, Rinehart and Winston, New York. ISBN: 0030851556, 9780030851551.