Paper No. : 04
Paper Title: Unit Operations in Food Processing
Module-06: Heat Transfer 2: Heat Convection

6.1 Introduction

Convection is a mode of heat transfer between a solid surface and the adjacent fluid in motion. Convection het transfer involves both the molecular movement and the bulk transport of the fluid. Higher the velocity of fluid more will be the rate of heat transfer. One common example is the boiling of water in a pan. The heat transfer takes place between the pan surface and the liquid layer adjacent to the surface. Had the water in the pan be still, the mode of heat transfer would have been by conduction.

Mathematical prediction of convective heat transfer is very difficult unlike the conduction and radiation heat transfer. So, the relationships between the variables are better explained with experimental results rather than the theoretical interpretations.

6.2 Newton's law of cooling

Newton experimentally proved that rate of cooling of a surface depends on the temperature difference between the surface of solid and that of the cooling fluid adjacent to it (Fig 6.1). The generalised equation of heat transfer is applied to express the Newton's law as

$$q = h_s A(T_s - T_\infty) \qquad \ldots (6.1)$$

Where, q is the rate of heat transfer (W), h_s is called surface heat transfer coefficient or film coefficient (W/m^2K), A is the exposed surface area to the liquid, T_s is the temperature of solid surface and T_∞ is the bulk temperature of liquid in motion. h_s here acts as a conductor of heat transfer in a hypothetical surface film. Comparing the conductance of conduction and convection heat transfer we get

$$h_s = \frac{k}{\Delta x} \qquad \ldots (6.2)$$

One important interpretation we get here, heat transfer coefficient depends on the thermal conductivity of fluid. Furthermore, since the convection includes the moving fluid the frictional resistance i.e. the viscosity and the fluid velocity affect the heat transfer coefficient.

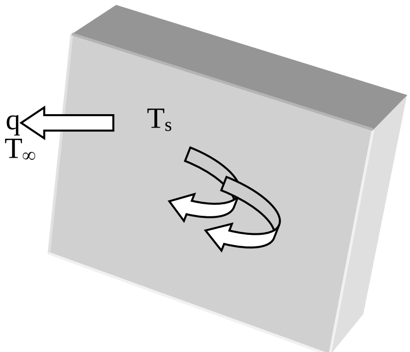

Fig.6.1 Convection heat transfer on the flat surface

6.3. Convection types

In convection, if the bulk movement of fluid is caused by buoyancy effect induced due to density difference as a result of temperature variation within the fluid mass, the convection is called natural convection. Boiling of water in a pan is one common example of natural convection. On the other hand, if fluid movement is caused by an external means like pump, fan or the wind, the type of convection is called forced convection. It is obvious that heat transfer in forced convection will be better compared to natural convection.

6.3.1 Natural convection

There are many examples of natural convection in food processing like cooling of cans taken out from a retort exposed to the ambient air, placing some food material in an oven without fan etc. Since, the convection heat transfer is a surface phenomenon that takes place at the interface between the fluid and solid, the controlling factors of heat transfer would be the fluid properties such as density, viscosity, thermal expansion, specific heat at constant pressure and thermal conductivity. The dimensions of surface like diameter and length will affect the heat transfer. The temperature difference between the surface and bulk fluid is the main driving force in convection.

As has been told earlier, experimental results are used to describe the relationship of these factors on heat transfer rate. For simplification, the factors are grouped as different dimensionless numbers named after eminent scientists. These numbers are

Nusselt number, $N_{Nu} = \frac{q_{convection}}{q_{conduction}} = \frac{h_s A \Delta T}{k A \Delta T / l} = \frac{h_s l}{k} \;\; or, \frac{h_s D}{k}$

Prandtl number, $N_{Pr} = \frac{momentum\ molecular\ diffusivity}{heat\ molecular\ diffusivity} = \frac{Kinematic\ viscosity}{thermal\ diffusivity} = \frac{\mu/\rho}{k/\rho c_p} = \frac{\mu c_p}{k}$

Grashof number, $N_{Gr} = \frac{buoyance\ force}{viscous\ force} = \rho^2 D^3 \beta g \, {}^{\Delta T}\!/_{\mu^2}$

The Nusselt number describes about the relative dominance of convection over conduction. When air is blown over a heated surface, the rate of heat transfer increases as a result of increased in Nusselt number. Prandtl number is the ratio of hydrodynamic layer and thermal boundary layer. The thicker the hydrodynamic layer less will be heat transfer and more the thermal boundary layer more will be the heat transfer. Therefore, less the Prandtl number better will be the rate of heat transfer. The Prandtl number for gas is about 0.7 and for water it is around 10.

The power function developed to relate the unit less numbers and the length ration is

$$N_{Nu} = K(N_{Pr})^a (N_{Gr})^b \left({}^{l}\!/_{D}\right)^c$$

K, a, b and c are the proportionality constants. The values of these constants are determined from the experimental results and eventually the Nusselt number is calculated. From the Nusselt number we find out the value of heat transfer coefficient. This value then put to equation (6.1) to calculate rate of heat transfer by convection.

6.3.1.1 Natural convection in vertical wall and cylinder

For $10^4 < (N_{Pr} N_{Gr}) < 10^9$, $N_{Nu} = 0.53(N_{Pr} N_{Gr})^{0.25}$ ... (6.3)

For $10^9 < (N_{Pr} N_{Gr}) < 10^{12}$, $N_{Nu} = 0.12(N_{Pr} N_{Gr})^{0.33}$ ... (6.4)

In case of air as the fluid medium, the above equations can be approximated as

$$h_s = 1.3(\Delta T / l)^{0.25}$$

$$h_s = 1.8(\Delta T)^{0.25}$$

6.3.1.2 Natural convection in horizontal cylinder

In case of laminar flow and the range $10^3 < (N_{Pr}N_{Gr}) < 10^9$, $N_{Nu} = 0.54(N_{Pr}N_{Gr})^{0.25}$ ... (6.5)

In case of air, for $10^4 < (N_{Pr}N_{Gr}) < 10^9$, $h_s = 1.3(\Delta T/l)^{0.25}$ ... (6.6)

for $10^9 < (N_{Pr}N_{Gr}) < 10^{12}$, $h_s = 1.8(\Delta T/l)^{0.33}$ ... (6.7)

6.3.1.3 Natural convection in horizontal plane

The empirical equations (6.5), (6.6) and (6.7) can be used for horizontal plane but with the replacement of diameter with characteristic length. In all the calculations the fluid properties are measured at film temperature rather than the bulk temperature of fluid.

6.3.2 Forced convection

The movement of fluid past a solid surface is caused by external forces in forced convection heat transfer. One example of forced convection heat transfer in food processing is blast freezer. Since the fluid is constantly drawn from the solid surface the rate of heat transfer is more compared to conduction heat transfer. Different cases of forced convection are discussed as follows.

6.3.2.1 Heat transfer inside pipe

For a long tube pipe with moderate temperature difference with laminar flow condition, the Nusselt number is found to be, $N_{Nu} = 4$.

In turbulence flow condition and $N_{Pr} > 0.5$

$$N_{Nu} = 0.023(N_{\mathrm{Re}})^{0.8}(N_{\mathrm{Pr}})^{0.4} \quad \dots (6.8)$$

For a more viscous fluid, the heat transfer depends on the fluid viscosity and its relation to different temperature conditions. In this case for $N_{\mathrm{Re}} > 10000$

$$N_{Nu} = 0.027\left({}^{\mu}/_{\mu_s}\right)^{0.14}(N_{\mathrm{Re}})^{0.8}(N_{\mathrm{Pr}})^{0.33} \quad \dots (6.9)$$

Where, the fluid properties are based on the bulk temperature except μ_s which the viscosity of fluid at solid surface temperature. For gases equation (6.9) can be reduced to

$$N_{Nu} = 0.02(N_{\mathrm{Re}})^{0.8} \quad \dots (6.10)$$

The fluid properties are measured at bulk temperature.

6.3.2.2 Heat transfer over plane surface

The Reynold's number uses the pipe length (*l*) instead of diameter of pipe *D* in its calculation. The equation is

$$N_{Nu} = 0.036(N_{\mathrm{Re}})^{0.8}(N_{\mathrm{Pr}})^{0.33} \quad for\ N_{Re} > 20000 \quad \dots (6.11)$$

Equation (6.11) can be simplified for gases as

$$h_s = 5.7 + 3.9v \quad for\ v < 5\ m/s$$

$$h_s = 7.4\ (v)^{0.8} \quad for\ 5 < v < 30\ m/s$$

These equations apply for smooth surfaces. The rough surface helps in maintaining turbulence to some extent, so the rate of heat transfer in rough surface would be more.

6.3.2.3 Heat transfer outside pipe

For gases and for liquid at moderate Reynold's number

$$N_{Nu} = 0.26(N_{\mathrm{Re}})^{0.6}(N_{\mathrm{Pr}})^{0.3} \qquad \dots (6.12)$$

For liquid at low Reynold's number ($N_{\mathrm{Re}} < 200$)

$$N_{Nu} = 0.86(N_{\mathrm{Re}})^{0.43}(N_{\mathrm{Pr}})^{0.3} \qquad \dots (6.13)$$

6.4 Overall heat transfer coefficient

In many heat transfer operations conduction and convection occur simultaneously. For example in shell and tube heat exchanger the steam condenses in shell and heat is given off to the outside tube by convection. The heat from outside surface to inside surface is by conduction and from inside surface to the liquid inside the tube is by means of convection (Fig. 6.2). If we represent the heat transfer in term of resistance, it will be

$q = \frac{\Delta T}{R_{total}}$ where, R_{total} is the resistance of conduction and convection both.

For the said example, $R_{total} = R_{convetion\ outside} + R_{conduction} + R_{convetion\ inside}$

According the fig (6.1) depicted here

$$R_{convetion\ outside} = \frac{1}{h_o A_0}$$

$$R_{convetion\ inside} = \frac{1}{h_i A_i}$$

$$R_{conduction} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k l}$$

So, $q = \frac{T_o - T_i}{\frac{1}{h_o A_0} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k l} + \frac{1}{h_i A_i}}$ ...... (6.14)

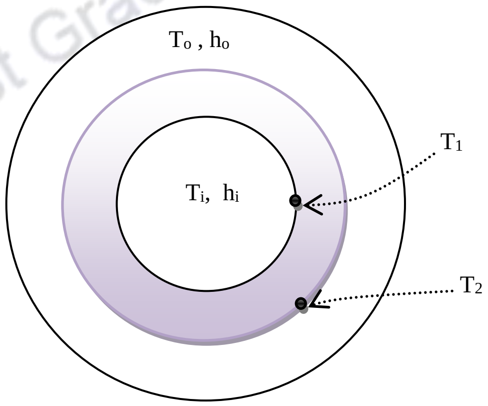

Fig. 6.2 Combined conduction and convection heat transfer in cylinder

The rate of heat transfer can also be represented as

$q = U_i A_i (T_o - T_i)$

Or, $q = U_o A_o (T_o - T_i)$

Where *U* is the overall heat transfer coefficient which takes care of both thermal conductivity and heat transfer coefficient.

Combining the above two expressions with equation (6.14) we get

$$\frac{1}{U_i A_i} = \frac{1}{h_o A_0} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k l} + \frac{1}{h_i A_i} \quad \text{or, } \frac{1}{U_i} = \frac{A_i}{h_o A_0} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k l} + \frac{1}{h_i} \qquad \text{... (6.15)}$$

And,

$$\frac{1}{U_o A_o} = \frac{1}{h_o A_0} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k l} + \frac{1}{h_i A_i} \quad \text{or, } \frac{1}{U_o} = \frac{1}{h_o} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k l} + \frac{A_0}{h_i A_i} \qquad \text{... (6.16)}$$

Overall heat transfer coefficient can be calculated based on the inside diameter using equation (6.15) or based on outside diameter by using equation (6.16).

6.5. Special case of convection heat transfer

6.5.1 Combined conduction and convection heat transfer

Consider a slab, for example the wall of a incubation chamber. The heat inside the room transfer through air to the inside wall by convection, then from inner wall to the outside wall through conduction and from outer wall to the atmosphere again by convection as shown in fig. (6.3).

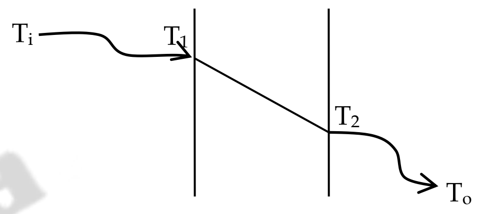

Fig.6.2 Combined conduction and convection heat transfer in slab

Heat transfer inside the room $q = h_i A(T_i - T_1)$

Heat transfer through the wall $q = \frac{(T_1 - T_2)}{\Delta x / kA}$

and heat transfer to the atmosphere $q = h_o A(T_2 - T_o)$

The thermal resistance is additive and the heat transfer in all the three cases will be same. So, combining all the three equations we get

$$q/_A = \frac{T_o - T_i}{\frac{1}{h_i} + \frac{\Delta x}{k} + \frac{1}{h_o}} \qquad \text{... (6.17)}$$

The overall heat transfer coefficient in this case

$$\frac{1}{U} = \frac{1}{h_i} + \frac{\Delta x}{k} + \frac{1}{h_o} \qquad \text{... (6.18)}$$

6.5.2 Heat transfer from condensing vapour

The condensation is the process of phase transfer from vapour to liquid by giving up the latent heat. Condensation in steam pipeline is an important aspect in food processing to look up for better understanding. If the condensed liquid forms a film over the condensing surface, it is called *film condensation*. The *drop-wise condensation* is one where the condensed liquid drops from the condensing surface without spreading on the surface as film. In drop-wise condensation the surface is exposed, thus the rate of heat transfer is more compared to film condensation, may be up to 10 times higher.

The empirical equation for condensation on vertical tubes or plane surfaces is

$$h_{sv} = 0.94[(k^3 \rho^2 g/\mu) \times (\lambda / l\Delta T)]^{0.25}$$

where λ $(\frac{J}{kg})$ is the latent heat of condensation, *l* is the height of plate or tube.

For condensation on a horizontal tube $h_{sh} = 0.72[(k^3\rho^2 g/\mu) \times (\lambda/D\Delta T)]^{0.25}$

where D is the diameter of tube.

6.5.3 Heat transfer to boiling liquid

Heating of liquid in a pan causes boiling of liquid. The rate of heat transfer depends on the temperature difference between the liquid and solid surface. If the difference is more than 20^0C, the heat transfer coefficient decreases because a layer of bubbles formed on the surface of solid covering the surface, thus reduced h_s. For temperature difference from 1 to 20 ^{0}C, values for h_s increases from 1200 to 60000 W/m^{2}K

$$h_s = 50(\Delta T)^{2.5}$$

Sometimes, the rate of heat transfer varies significantly with the resistances offered and not by the change of heat transfer coefficient.

6.6 Critical thickness of insulation

Insulating materials are used to reduce heat loss from the fluid flowing inside a pipe. By adding insulating material the thermal resistance increases, but at the same time the outer diameter of pipe increases which may increase the heat loss. We cannot ascertain whether the heat transfer increase or decrease with the addition of insulating materials. So, a critical diameter is derived beyond which if we add insulation, the heat transfer will decrease. In other words, the minimum thickness given to any heating surface for effective control of heat loss is the critical thickness.

For cylinder the critical thickness is $r_{cr} = \frac{k}{h_o}$

and for sphere, the value is $r_{cr} = \frac{2k}{h_o}$

References

1. *Transport Processes and Unit Operations* (3rd Edition), C. J. Geankoplis, Prentice Hall Inc. Publ., 1993.
2. *Unit operations in food processing*, R.L. Earle and M.D. Earl, NZIFST (Inc.) Publ., 1983.