Paper No. : 04
Paper Title: Unit Operations in Food Processing
Module- 05: Heat Transfer 1: Heat Conduction

5.1 Introduction

Food engineers very often use heating and cooling processes which fall under one unit operation called heat transfer. Heat transfer occurs in various food processing operations like evaporation, drying, freezing, refrigeration, heat sterilization, pasteurization etc. Basic understanding on the principles of heat transfer is inevitable for food engineers either to design thermal process equipments, their control for better performance or to modify the existing process design.

5.2 Heat transfer theory

One generalized equation of any transfer process is,

$$rate\ of\ transfer = \frac{driving\ force}{resistance}$$

For heat to transfer from one body to other there must be some temperature difference between the two bodies and the medium must allow the heat to transfer through it. Temperature difference is the driving force and the resistance offered by the medium against the flow of heat is the resistance. The generalized equation can be reframed for heat transfer as,

$$rate\ of\ heat\ transfer = \frac{temperature\ difference}{resistance\ by\ the\ medium}$$

During processing, temperature of the bodies may change and hence the driving force. The heat transfer process where the change of temperature occurs, the process is called unsteady state heat transfer. One example of unsteady state heat transfer is the heating and cooling of cans in retorts. Unsteady state heat transfer calculations are more complex than steady state where temperature does not change.

There are various mechanisms of heat transfer viz. conduction, convection and radiation. In thermal processes the heat transfer occurs either by one mechanism or in combination of two or three mechanisms. But of course, one mode of heat transfer plays a dominating role. Design calculations are carried out based on the dominating mechanism of heat transfer neglecting the others if they are not significant.

a) Conduction: Heat transfer occurs because of the transfer of vibrational energy from one molecule to the adjoining molecules closely associated in a solid mass. Free electrons in molecular level might carry the thermal energy and electrical energy with in a system. Physical movement of body does not take place in conduction.
   Example:- heating of metal body
b) Convection: Convection is associated with fluids since heat transfer in a system occurs due to the mass movement within the system.
   Example:- boiling of water
c) Radiation: Radiation is the transfer of thermal energy in the form of electromagnetic spectrum in vacuum. Generation of electromagnetic spectrum occurs at high temperature range.
   Example:- Microwave heating, heating of bread crust inside baking oven

5.3 Conduction

In case of conduction rate of heat transfer is *dQ/dt*, the driving force is the temperature difference per unit length *dT/dx* and the resistance offered by the medium is 1/kA. The reciprocal of resistance is conductance (kA).
So, the equation of heat transfer will be,

$$\frac{dQ}{dt} = -\frac{dT/_{dx}}{1/_{kA}} \quad or, \;\; q_x = -kA\frac{dT}{dx} \qquad \ldots. \; (5.1)$$

Where, A is the area of cross section perpendicular to the direction of heat flow, k is the thermal conductivity of the body. The unit of heat transfer rate q is *J/s* or Watt (*W*) and that of thermal conductivity is *W/mK* or W/m^0C.

Equation (5.1) is known as **Fourier's equation** for heat conduction.

Note: The flow of heat occurs from a hot body to cold body i.e. in negative temperature gradient. Thus a minus sign appears in the Fourier equation.

5.4 Thermal conductivity

Thermal conductivity can be measured using Fourier's equation of heat conduction. The thermal conductivity gives the idea of a substance's behaviour in transferring heat. More the thermal conductivity better will be the rate of heat transfer. Though a slight change in thermal conductivity happens with a change in temperature, it is considered to be a constant thermal property of a substance.

Thermal conductivity depends on the molecular arrangement of a substance. The metals have high thermal conductivity in the range of 50-400 *W/mK*. Liquids have relatively low thermal conductivities and for gases the values are even less. Food products contain water as the main composition whose thermal conductivity is 0.7 *W/mK*. So, the thermal conductivity of food materials varies from 0.6-0.7 *W/mK*. The molecules of ice are closely packed compared to that of water. So, thermal conductivity of ice is in higher sight, about 2.3 *W/mK*. For this reason, the thermal conductivity of frozen foods is higher at normal temperature. The insulating materials like thermo-cool, cardboard etc. contain air pockets in their matrix. Since, thermal conductivity of air is very less about 0.024 *W/mK*, the thermal conductivity of insulating materials such as rubber cork, foamed plastics etc. varies in the range of 0.03-0.06 *W/mK*.

5.5 Conduction through a slab

Let us consider a uniform slab as shown in fig (5.1). The flow of heat is in x-direction. The temperature of left side face perpendicular to X-axis is T_1 ^{0}C and that of right face is T_2 ^{0}C. The temperature T_1 is more than T_2. The thickness of the slab is x m. According to Fourier's law,

$$q = \frac{dQ}{dt} = -kA\frac{dT}{dx}$$

Or, $$q = -kA\frac{(T_1 - T_2)}{(x_1 - x_2)}$$

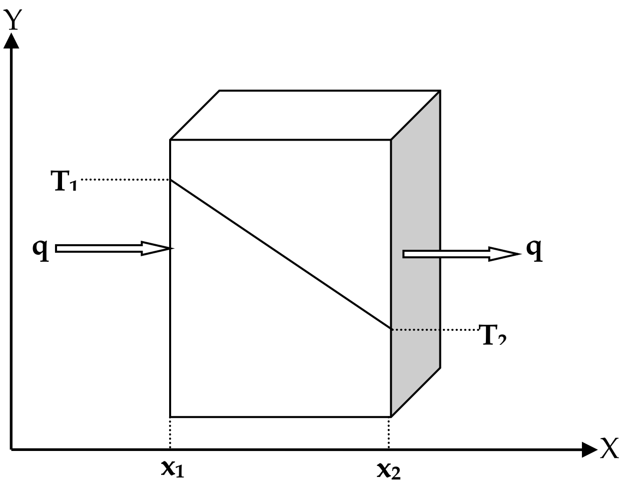

Fig.5.1 Conduction heat transfer through uniform slab

Or, $q = k\,A\,\frac{(T_1 - T_2)}{(x_2 - x_1)} = \left(\frac{k}{x}\right)\,A\,\Delta T \quad \ldots \quad (5.2)$

The term *k/x* is called heat conductance whose unit is W/m^2K. Equation (5.2) is considered to be the basic equation to calculate heat transfer in uniform walls.

Problem 5.1 Determine thermal conductivity of an insulating material of flat slab 25 mm thickness whose temperatures at both sides of the slab are 45 and 30 ^{0}C. The heat flux measured is 35 W/m^2.

Solution: The term q/A is called heat flux. Rearranging equation (5.2)

$k = \frac{q}{A}\frac{x}{\Delta T}$

$\Rightarrow k = 35\,\frac{0.025}{15} = 0.058\,W/mK$

5.6 Heat conduction in series

One case of heat transfer is through a series of composite walls, like in the case of wall of cold storage room where the layers of brick, cork, plastic, wood etc. are used to make an insulating wall. Consider a wall of three composite layers as shown in fig. (5.2). Equation (5.2) is applied in each layer to find out the total heat transfer.

If steady state transfer is assumed, the rate of heat transfer in all the layers will be same.

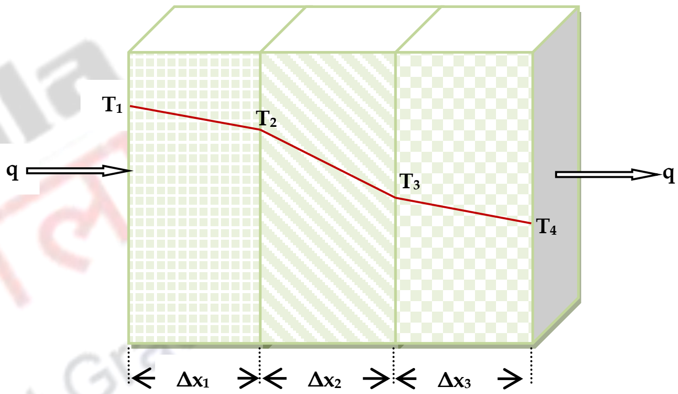

Fig. 5.2 Conduction heat transfer in series

$q = k_1\,A_1\,\frac{(T_1 - T_2)}{\Delta x_1} = k_2\,A_2\,\frac{(T_2 - T_3)}{\Delta x_2} = k_3\,A_3\,\frac{(T_3 - T_4)}{\Delta x_3} \quad \ldots \quad (5.3)$

If the areas are same in all the three layers, $A_1 = A_2 = A_3 = A$

The equation (5.3) thus will be,

$q/_A = \frac{(T_1 - T_2)}{\Delta x_1/_{k_1}} = \frac{(T_2 - T_3)}{\Delta x_2/_{k_2}} = \frac{(T_3 - T_4)}{\Delta x_3/_{k_3}}$

Or, $q/_A = \frac{\Delta T_1}{R_1} = \frac{\Delta T_2}{R_2} = \frac{\Delta T_1}{R_3}$

But, $\Delta T_1 + \Delta T_2 + \Delta T_3 = \Delta T = (T_1 - T_4)$

So, $q/_A = \frac{T_1 - T_4}{R_1 + R_2 + R_3}$

Since, $R_1 + R_2 + R_3 = \frac{1}{U}$

$q = UA\Delta T \quad ….. \quad (5.4)$

Where, U is called overall heat transfer coefficient. The unit is W/m^2K. Equation (5.4) is the universal equation which can also be used in other modes of heat transfer.

5.7 Conduction through cylindrical wall

The Fourier equation (5.1) for cylindrical wall can be expressed as,

$$q = -kA\frac{dT}{dr} \quad .... (5.5)$$

Where, r is the radius of wall surface and A is the curved surface area through which heat flows.

Since, area $A = 2\pi rl$ where, l is the length of pipe.

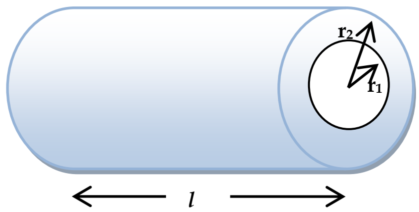

Fig.5.3 Conduction through a cylindrical wall

Equation (5.5) will be, $q = -k(2\pi rl)\frac{dT}{dr}$

Rearranging the above equation

$$q\frac{dr}{r} = -k(2\pi l)dT$$

Integrating above equation with boundary condition $r = r_1$ at $T = T_1$ and $r = r_2$ at $T = T_2$, we get,

$$q\int_{r_1}^{r_2}\frac{dr}{r} = -2\pi kl\int_{T_1}^{T_2} dT$$

$$\Rightarrow \quad q\, ln\left(\frac{r_2}{r_1}\right) = -2\pi kl(T_2 - T_1)$$

$$\Rightarrow \quad q = \frac{2\pi kl(T_1 - T_2)}{ln\left(\frac{r_2}{r_1}\right)} \quad .... \quad (5.6)$$

Multiplying numerator and denominator with $(r_2\text{-}r_1)$ in equation (5.6)

$$q = k\frac{2\pi r_2 l - 2\pi r_1 l}{ln\left(\frac{2\pi l r_2}{2\pi l r_1}\right)}\frac{(T_1 - T_2)}{r_2 - r_1}$$

$$\Rightarrow \quad q = k\frac{A_2 - A_1}{ln\left(\frac{A_2}{A_1}\right)}\frac{(T_1 - T_2)}{r_2 - r_1}$$

Or, $q = kA_{lm}\frac{(T_1 - T_2)}{r_2 - r_1}$ .... (5.7)

Where A_{lm} is log mean area.

Problem 5.2 A steel pipe of inner diameter 5 cm and outer diameter 10 cm is used to transport steam. If the inside and outside wall temperatures are 90 ^{0}C and 40 ^{0}C respectively, find out the amount of heat lost to atmosphere per unit length. Assume thermal conductivity of steel pipe k=45 *W/mK.*

Solution: Given data

$$r_1 = \frac{5\times10^{-2}}{2} = 2.5\times10^{-2}\,m \;, \qquad r_2 = \frac{10\times10^{-2}}{2} = 5\times10^{-2}\,m$$

$$T_1 - T_2 = (90 - 40)°C = 50\,°C \,,\; k = 45\,\frac{W}{mK}$$

Using equation (5.6)

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$$q = \frac{2\pi kl(T_1 - T_2)}{ln\left(\frac{r_2}{r_1}\right)} = \frac{2\pi \times 45 \times 1 \times 50}{ln\left(\frac{5}{2.5}\right)}$$

$$= 20391.77\ W = 20.39\ kW$$

The heat loss to the environment will be *20.39 kW* per meter length of pipe.

5.8 Plane wall with internal heat generation

In certain systems heat is generated inside the conducting medium. The biological material compost is one such system where heat is generated. Fruits and vegetables also respire to generate some amount of heat in storage bags.

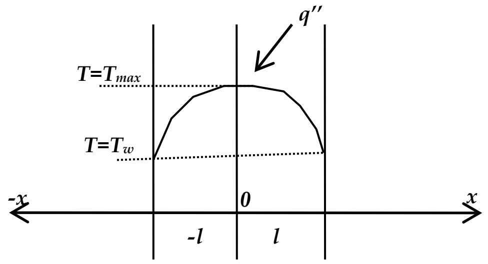

Fig.5.4 Internal heat generation in plane wall

It is assumed that the flow is one dimensional. Since heat is generated an unsteady state heat transfer occurs. Assuming zero accumulation and q'' amount of heat generated per unit volume, the equation will be,

$$q_x + q''(\Delta x A) = q_{x+\Delta x}$$

Now, for an infinitesimally small section of body, Δx approaches zero. Dividing by Δx we get,

$$\frac{q_x}{\Delta x} + q''(A) = 0 \quad \ldots (5.8)$$

Substituting equation (5.1) with (5.8) we obtain

$$\frac{d^2T}{dx^2} + \frac{q''}{k} = 0$$

The integration of the above equation gives

$$T = -\frac{q''}{2k}x^2 + C_1x + C_2 \quad \ldots. (5.9)$$

From the fig.5.4 we can see the boundary condition: at $x=0$, $T= T_{max}$ and at $x= l$ or $-l$, $T= T_w$

The equation (5.9) at boundary conditions will be

$$T = -\frac{q''}{2k}x^2 + T_{max}$$

$$\Rightarrow T_{max} = \frac{q''}{2k}l^2 + T_w \quad \ldots.. (5.10)$$

Equation (5.10) for heat generation in cylinder will be derived as

$T_{max} = \frac{q''}{2k}r^2 + T_w$, where, r is the radius of the cylinder.

References

1. *Fundamentals of food process engineering*, Romeo T. Toledo, Springer, 3rd edn., 2007
2. *Transport Processes and Unit Operations* (3rd Edition), C. J. Geankoplis, Prentice Hall nc. Publ., 1993.