Paper No. : 04

Paper Title : UNIT OPERATIONS IN FOOD PROCESSING

Module – 4 : Fundamentals of fluid flow theory

Introduction

Importance of fluid flow in food processing

- Study of fluid behaviour
- Liquid handling in processing plants
- Selection of pumps
- Pipeline design
- Selection of process equipments based on the properties of fluid

Science of fluid flow

Fluid Mechanics: The branch of engineering science that deal with fluid flow.

There are three branches of fluid mechanics

1) Fluid Statics:

   The study of fluid behaviour at rest.

2) Fluid kinematics:

   The study of flow of fluid with out considering its causes.

3) Fluid dynamics:

   It involves the study of flow and the forces causing the flow.

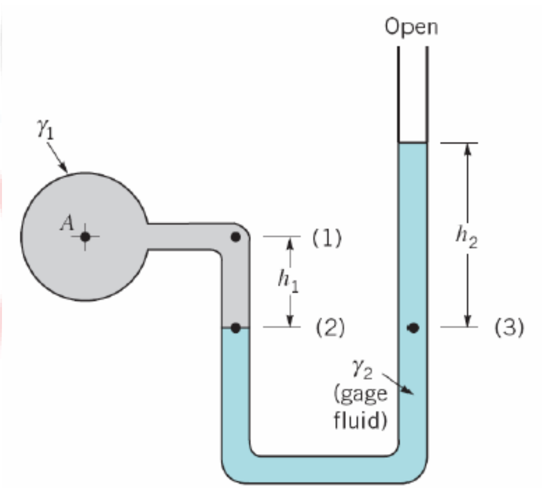
Measurement of pressure

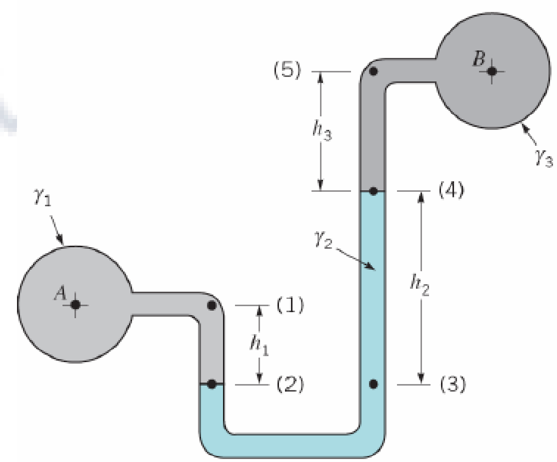
Devices

1) Manometers (Piezometer, simple U-tube and differential manometers)
2) Mechanical gauges (Bourdon's gauge and diaphragm)

Manometers are the devices that measure the pressure of fluid at a point which is equal to the mass of standing fluid above it. The pressure difference at two points is measured by ***balancing the column*** *of same liquid or other liquid.*

Manometric fluid: mercury





$$p_A + \gamma_1 h_1 - \gamma_2 h_2 = 0$$

$$p_A = \gamma_2 h_2 - \gamma_1 h_1$$

$$p_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = p_B$$

$$p_A - p_B = \gamma_2 h_2 + \gamma_3 h_3 - \gamma_1 h_1$$

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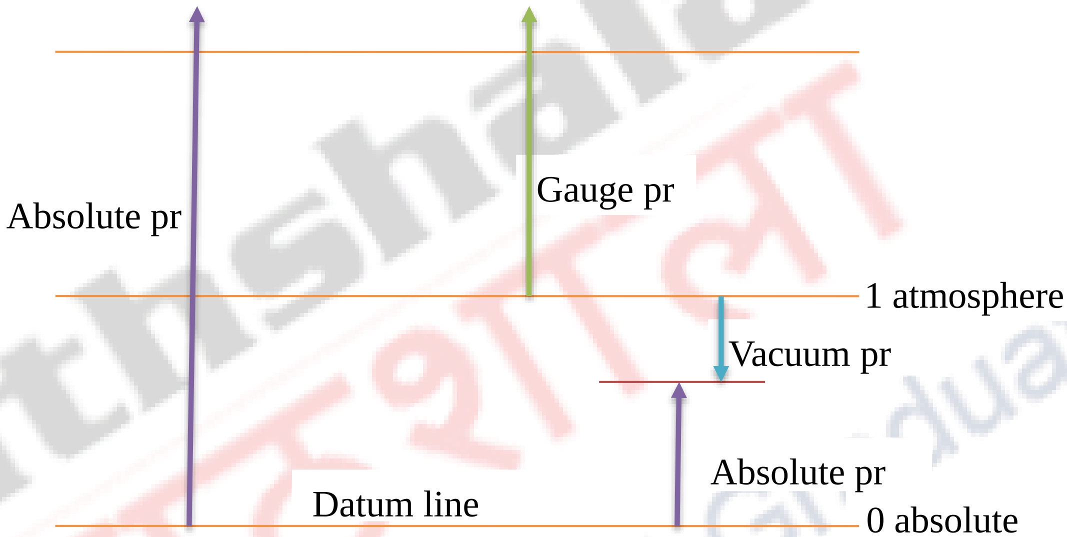


Absolute pressure: The pressure measured w.r.t. absolute vacuum pressure

Gauge pressure: The pressure measured above atmospheric pressure

Vacuum pressure: The pressure in short of atmospheric pressure

$$P_{abs} = P_{atm} + P_{gauge}$$

$$P_{vacuum} = P_{atm} - P_{abs}$$

Fluid Kinematics

Fluid kinematics: *A branch of fluid mechanics, which describes the fluid motion and it's consequences without considering the causes of fluid motion.*

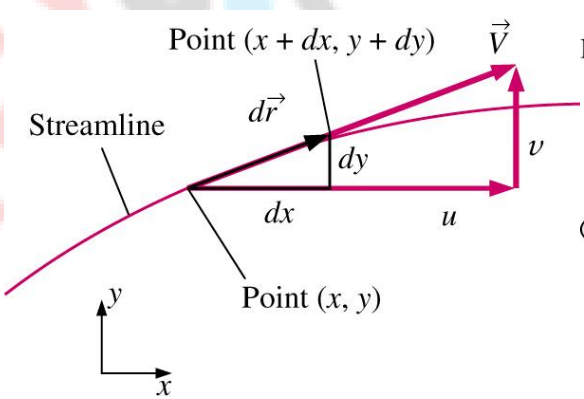
Flow patterns

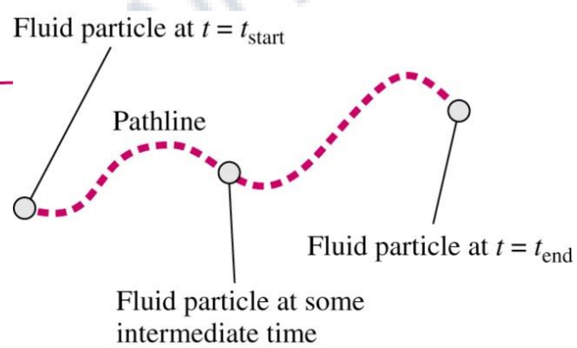
Stream line: An imaginary line in the flow field whose tangent to the line at any point is the direction of local velocity at that point.

Path line: Actual path travelled by an individual fluid particle

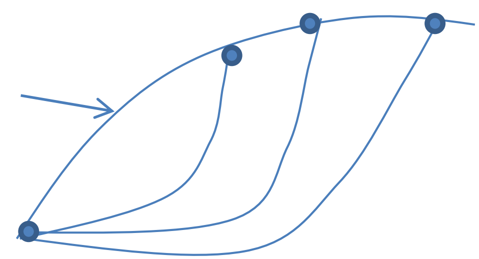
Streak line: The line formed by all fluid particles that have passed through a given fixed point

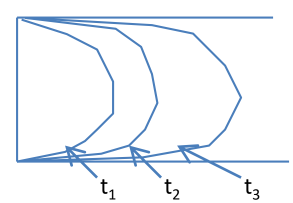
Time line: A set of points form a definite line at any instant in flow field





Streak line





Time line

Types of Flow

Steady state flow: $\left(\frac{\partial V}{\partial t}\right)_{xyz} = 0$ $\left(\frac{\partial \rho}{\partial t}\right)_{xyz} = 0$

Un-steady state flow: $\left(\frac{\partial V}{\partial t}\right)_{xyz} \neq 0$ $\left(\frac{\partial \rho}{\partial t}\right)_{xyz} \neq 0$

Uniform flow: $\left(\frac{\partial V}{\partial S}\right)_{t} = 0$

Non-uniform flow: $\left(\frac{\partial V}{\partial S}\right)_{t} \neq 0$

Compressible flow: density changes with time or space

Laminar flow: A set of points flows through a well defined path or streamline

One-two and three dimensional flow: $\vec{V} = \vec{u}\hat{\imath}$

$$\vec{V} = \vec{u}\hat{\imath} + \vec{v}\hat{\jmath}$$

$$\vec{V} = \vec{u}\hat{\imath} + \vec{v}\hat{\jmath} + \vec{w}\hat{k}$$

Rotational flow: Fluid flows in streamline and it rotates about its own axis.

Irrotational flow: Fluid does not rotate about its own axis

\eynold's . umber

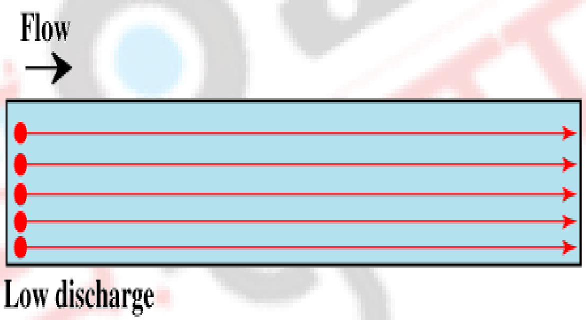
$$N_{Re} = \frac{inertial force}{viscous force} = \frac{\rho A v^2}{\mu \frac{v}{l} \times A} = \frac{\rho l v}{\mu}$$

Or, $N_{Re} = \frac{\rho d V}{\mu}$ for pipe flow

Reynold's no. can also be represented in terms of mass flow rate.

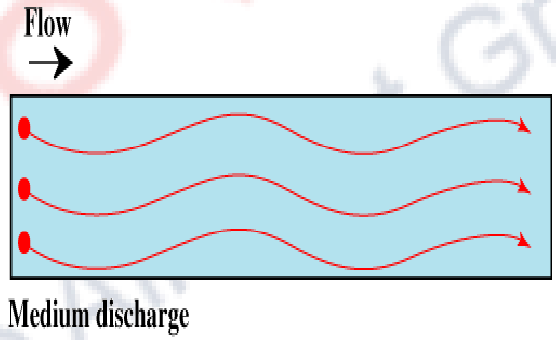
$$m = \rho A v$$

So, $N_{Re} = \frac{4m}{\pi d \mu}$



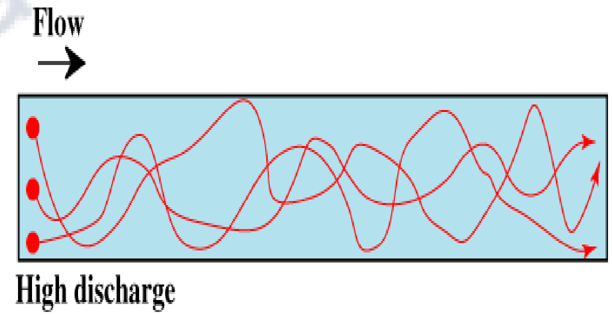
Laminar Flow

$N_{Re} < 2100$



Transitional Flow

$2100 < N_{Re} < 4000$

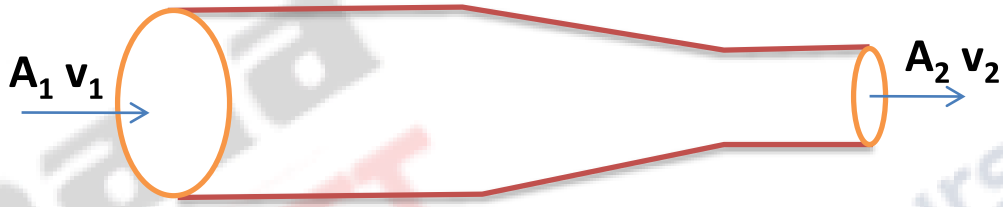


Turbulent Flow

$N_{Re} > 4000$

quation of continuity

Principle: Conservation of mass



Let consider two points 1-1 and 2-2 in a pipe line of different cross section

Mass flow rate also known as discharge.

Mass flow rate at 1-1 $m_1 = \rho_1 A_1 V_1$

Mass flow rate at 2-2 $m_2 = \rho_2 A_2 V_2$

According to the law of conservation of mass $m_1 = m_2$

$$\therefore \quad \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

For incompressible fluid $\rho_1 = \rho_2$

$$\boxed{A_1 V_1 = A_2 V_2} \qquad V \propto \frac{1}{A}$$

So,
The product of cross sectional area and velocity is called discharge

$$\boxed{q_1 = q_2}$$

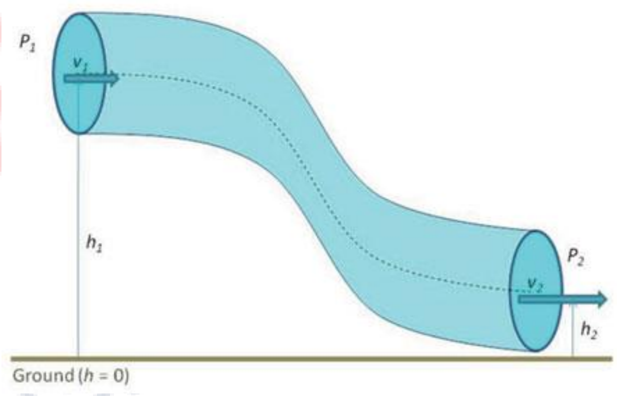
Fluid Dynamics

Bernoulli's Theorem

Bernoulli's theorem works with the principle of conservation of energy.

$$h + \frac{P}{\rho g} + \frac{v^2}{2g} = \text{Constant}$$

Elevation head (h), Pressure head ($\frac{P}{\rho g}$), Energy head ($\frac{v^2}{2g}$)

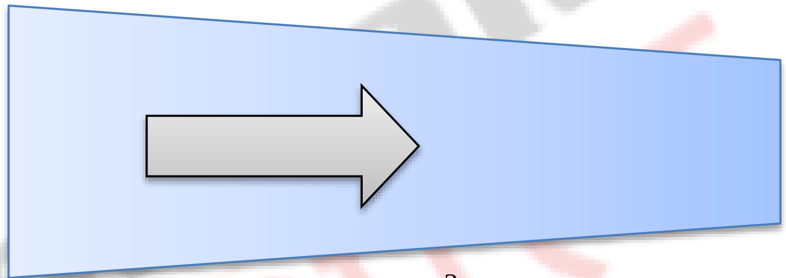


Assumptions:

1) Fluid is ideal and incompressible
2) Fluid flows in laminar region
3) Steady state flow
4) Flow is stream line

$$h_1 + \frac{P_1}{\rho g} + \frac{v_1^2}{2g} = h_2 + \frac{P_2}{\rho g} + \frac{v_2^2}{2g}$$

Milk is flowing at a rate of 1500 l/h through a pipe of diameter 10 cm at a pressure of 80 kPa. The pipe is reduced to 7 cm diameter at the other end. Neglecting friction loss in pipe and viscosity of liquid, find out the pressure built up at the end of the pipe. Assume milk density as 1030 kg/m³



$$q = 1500 l/h = \frac{1500\times10^{-3}}{3600} m^3/s = 4.17\times10^{-4} m^3/s$$

$$A_1 = \frac{\pi}{4}(0.1)^2 = 7.85\times10^{-3} m^2$$

$$A_2 = \frac{\pi}{4}(0.05)^2 = 2.50\times10^{-3} m^2$$

$$v_1 = \frac{4.17\times10^{-4}}{7.85\times10^{-3}} m/s = 0.053 m/s$$

$$v_2 = \frac{4.17\times10^{-4}}{2.50\times10^{-3}} m/s = 0.167 m/s$$

Applying Bernoulli's equation

$$\cancel{z_1} + \frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \cancel{z_2} + \frac{P_2}{\rho g} + \frac{v_2^2}{2g}$$

$$P_2 = P_1 + \frac{\rho}{2}\left(v_1^2 - v_2^2\right)$$

$$P_2 = 80 + \frac{1030}{2}\left(0.053^2 - 0.167^2\right) = 79.98 kPa$$

Energy losses in Pipes

When fluid moves through a pipe or fittings, it encounters some resistance due to which some energy is lost.

Energy losses

Major loss

- Friction loss

Minor losses

- Due to sudden enlargement
- Due to sudden contraction
- Due to bend in pipe
- Due to pipe fittings
- Due to obstructions in pipe

oss due to pipe friction

Assuming horizontal pipe and a constant velocity flow

$$h_1 + \frac{P_1}{\rho g} + \frac{v_1^2}{2g} = h_2 + \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_f$$

So, $$h_f = \frac{P_1 - P_2}{\rho g} \quad ...(1)$$

A friction factor (Darcy's friction factor, *f*) is developed to describe the frictional losses in pipe.

$$f = \frac{Drag\ \ force\ per\ wetted\ \ perimeter}{\rho \times \frac{v^2}{2}} = \frac{-\frac{dP}{dx}d}{\rho \times \frac{v^2}{2}}$$

$$\Rightarrow -\frac{dP}{dx} = \frac{f\rho v^2}{2d}$$

$$\Rightarrow P_1 - P_2 = \frac{fl\rho v^2}{2d} \quad ...(2)$$

Combining eqn 1 and 2 we get, $$h_f = f\frac{l}{d}\frac{v^2}{2g}$$ **Darcy- Weishbach eqn**

anning's friction factor and oody's

Darcy-Weishbach equation can also represented as

$$h_f = 4f'\frac{l}{d}\frac{v^2}{2g}$$

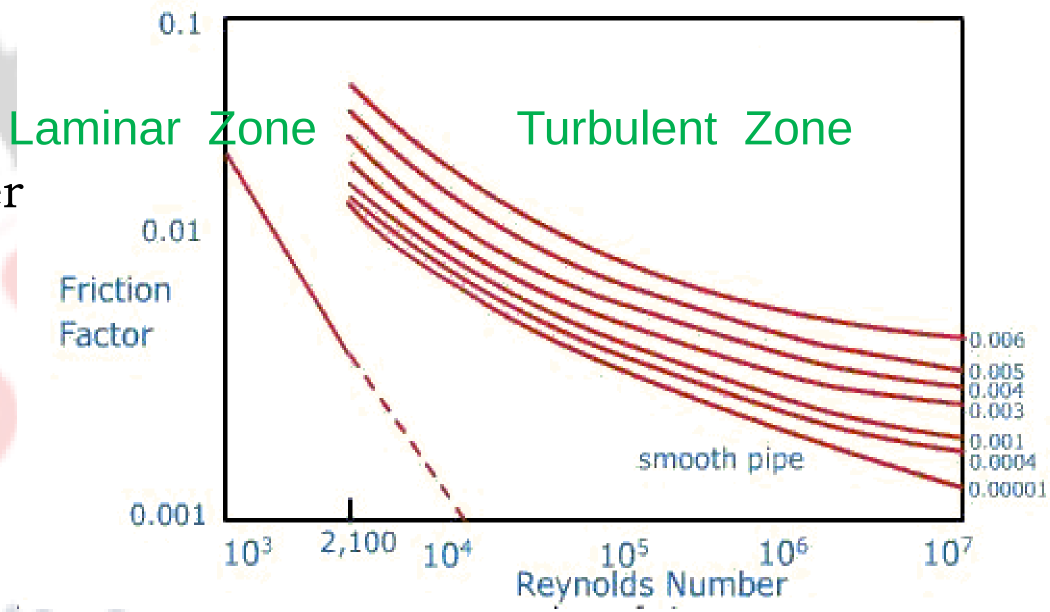
Where *d* in this case is diameter of pipe

In laminar region (N_{Re} <2100)

$$f' = \frac{16}{N_{Re}}$$

For 4000 < N_{Re} < 10^4

$$f' = \frac{0.079}{(N_{Re})^{0.25}}$$



From the Moody's diagram it is seen that Friction factor is a function of Reynold's no. and roughness of pipe.

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Loss due to sudden enlargement: $$h_e = \frac{(v_1 - v_2)^2}{2g}$$

Loss due to sudden contraction: $$h_c = k\frac{{v_2}^2}{2g} \qquad where, k = \left(\frac{1}{C_c} - 1\right)^2$$

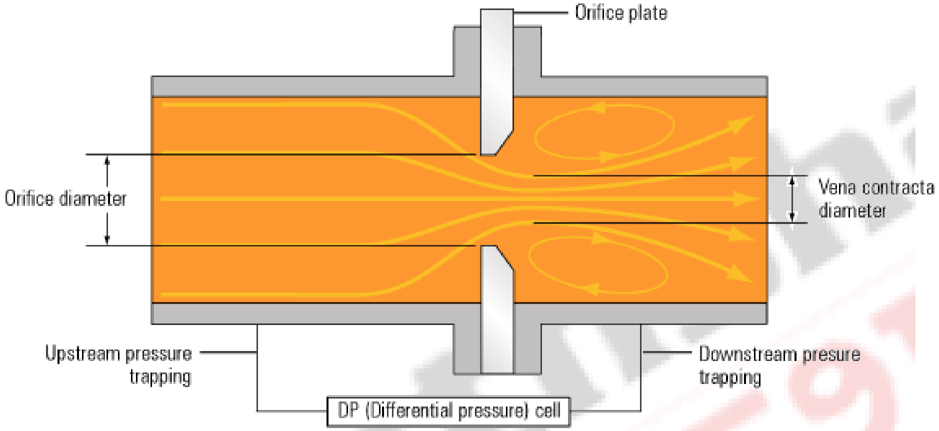
Loss due to bend in pipe: $$h_b = k\frac{v^2}{2g}$$

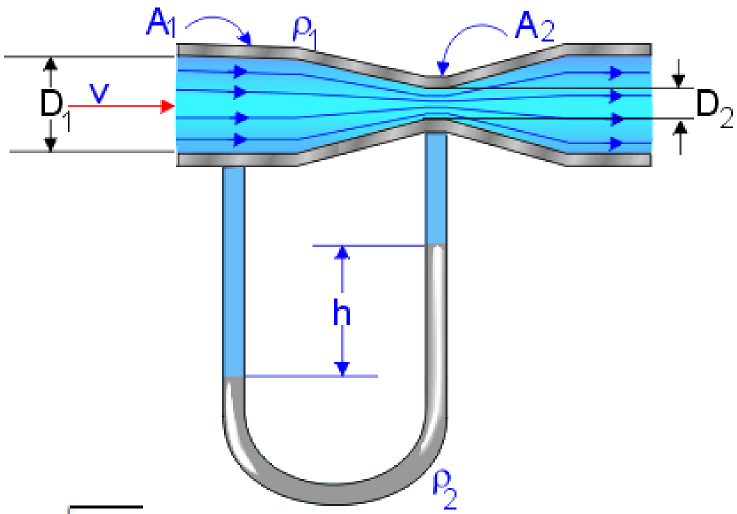
Loss due to fittings in pipe: $$h_{ft} = k\frac{v^2}{2g}$$

Loss at the entrance of pipe: $$h_i = 0.5\frac{v^2}{2g}$$

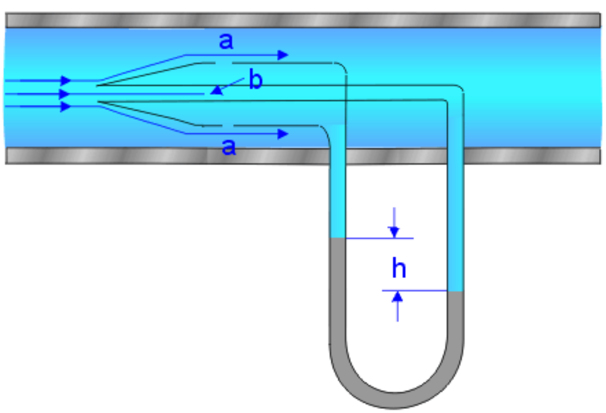
Loss at the exit of pipe: $$h_o = \frac{v^2}{2g}$$

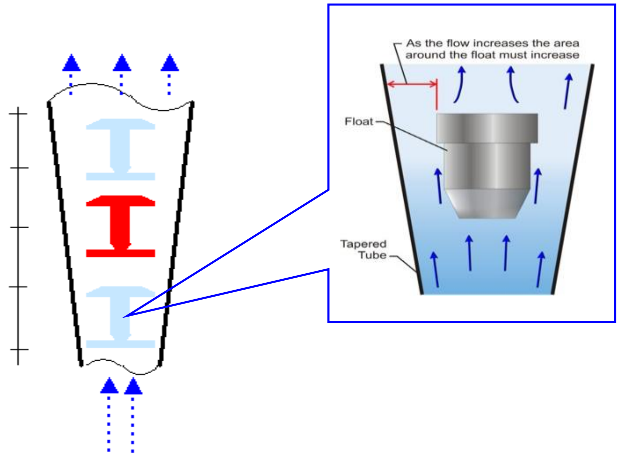
Flow measurements





$$q_{act} = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$





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