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TABLE OF CONTENTS

1. Learning Outcomes

After studying this module, you shall be able to

- Gain an insight about the oligopoly market structure
- Understand the determination of stable equilibrium in the classical duopoly models of Cournot and Bertrand
- Comprehend the Stackelberg's duopoly model of sophisticated behavior, which is an extension of the Cournot model

2. Introduction

Oligopoly is a market structure in which only a few firms compete with one another, and entry by new firms is impeded. The product that the firms produce may be homogenous (as in the case of steel) or differentiated (as with automobiles). Oligopoly is a prevalent form of market structure. Examples of oligopolistic industries include automobiles, steel, aluminium, petrochemicals, electrical equipment, and computers. If there are two sellers in the market, it is called a situation of duopoly.

Such a market structure is characterized by the following unique features—

- Since there are few rival firms, the behavior of the firms is mutually interdependent and strategic. This implies that the action of one influences the actions of others. The price and output policy of a firm will affect the price and quantity sold by other firms.
- Advertising is a powerful instrument in the hands of an oligopolist. A firm can take up aggressive advertising with the intention of capturing a large part of rivals market.
- Oligopoly has few firms dominating the market. These firms produce products that may be homogenous or differentiated, and also undertake measures to restrict entry of new firms.

Because of the high degree of interdependency among oligopolistic firms, diverse behavioural pattern may emerge. The firms may collude with each other and jointly agree to set prices and quantities that maximize the sum of their profits. This sort of behavior is called collusive behavior. Another form of interaction is when the firms compete with each other in one form or the other. Such a behavior is described as non-collusive behavior. Based on different behavioural assumptions, there are a variety of **collusive** and **non-collusive** models of oligopoly. Some of the non-collusive models include Bertrand's model, Cournot's model, Stackelberg's model, Chamberlin's model, and Paul Sweezy's model. Price leadership, Cartels are some of the collusive models.

The present and the subsequent modules will focus on the **non-collusive models** of oligopoly. In this module, we will restrict ourselves to the case of two firms, i.e., duopoly. The duopoly case allows us to capture many of the important features of firms engaged in strategic interaction without the notational complications involved in models with a larger number of firms.

3. Cournot's Duopoly Model

Cournot model is an oligopoly model in which firms produce a homogenous good, each firm treats the output of its competitors as fixed, and all firms decide simultaneously how much to produce.

3.1 Simple Duopoly model with zero costs

Consider a simple model of duopoly first developed by Augustin Cournot. Suppose there are two firms, each owning a spring of mineral water. It is assumed that it is produced at zero costs. Further, the firms sell their output in a market with a straight-line demand curve. Each firm acts on the assumption that its competitor will not change its output, and decides its own output so as to maximize profit.

Let us assume that firm A is the first to start producing and selling mineral water. As illustrated in Figure 3.1, this firm will produce quantity A, at price P where profits are at a maximum because at this point MC =- MR = 0. The elasticity of market demand at this level of output is equal to unity and the total revenue of the firm is at a maximum. With zero costs, maximum revenue implies maximum profits. Now firm B assumes that A will keep its output fixed (at OA), and hence considers that its own demand curve is CD′. It is evident that firm B will produce half the quantity AD′, because at this level of output (AB) and at price P′, its revenue and profit is at a maximum. Therefore, B's output is ¼ (= ½×½) of the total market.

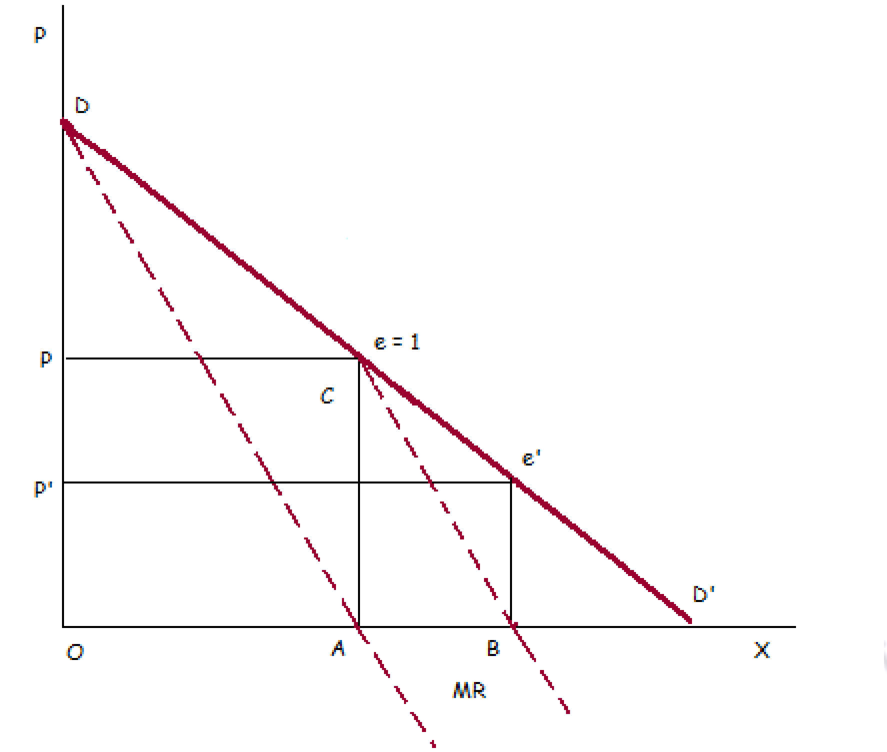

FIGURE 3.1: COURNOT DUOPOLY MODEL WITH ZERO COSTS

Firm A, faced with this situation, assumes that B will retain his quantity constant in the next period. So he will produce one-half of the market which is not supplied by B. Therefore, A will, in the next period, produce ½(1-¼) = ⅜ of the total market. Firm B reacts on the Cournot assumption, and will produce one-half of the unsupplied section of the market, i.e. ½(1-⅜).

This action-reaction pattern continues and eventually equilibrium is reached in which each firm produces one-third of the total market. Each firm maximizes its profit in each period, but the industry profits are not maximized. That is, the firms would have higher joint profits if they recognized their interdependence. This would lead them to act as a monopolist, producing one-half of the total market output, selling it at the profit-maximizing price P, and sharing the market equally, that is, each producing one-quarter of the total market (instead of one-third).

The Cournot solution is stable. Each firm supplies one-third of the market, at a common price which is lower than the monopoly price, but above the pure competitive price (which is zero in this case of costless production). It can be shown that if there are three firms in the industry, each will produce one-quarter of the market and all of them together will supply ¾ of the entire market OD̕. And, in general, if there are n firms in the industry each will provide 1/(n+1) of the market, and the industry output will be n/(n+1). The larger the number of firms, the closer is output and price to the competitive level.

3.2 Reaction-Curves Approach

The assumption of costless production in Cournot's model is however unrealistic. The reaction-curves approach allows the relaxation of the assumption of identical costs and identical demands without impairing the validity of the model.

This can be explained by considering a case of two firms producing a homogenous good. Each firm must decide how much to produce, and the two firms make their decisions simultaneously. Based on the Cournot assumption, each firm treats the output level of its competitor as fixed when deciding how much to produce. Suppose firm A thinks that firm B will produce nothing. In that case, firm A's demand curve is the market demand curve (given by D_1 in Figure 3.2). The corresponding marginal revenue curve is MR_1.

Assume that firm A's marginal cost MC_1 is constant. As shown in the figure, firm A's profit maximizing output is 50 units, the point where MR_1 intersects MC_1. So, if firm B produces zero, firm A should produce 50.

Now suppose firm A thinks firm B will produce 50 units. Then firm A's demand curve is D_2, and the corresponding marginal revenue curve is MR_2. Firm A's profit maximizing output is now 25 units ($MR_2 = MC_1$). Suppose firm A thinks that firm B will produce 75 units. Then firm A's demand curve is D_3, and the corresponding marginal revenue curve is MR_3. Firm A's profit maximizing output is now 12.5 units ($MR_3 = MC_1$). Finally, suppose firm A thinks that firm B will produce 100 units. Then firm A's demand and marginal revenue curves would intersect its marginal cost on the vertical axis. This is to say that firm A will not produce anything.

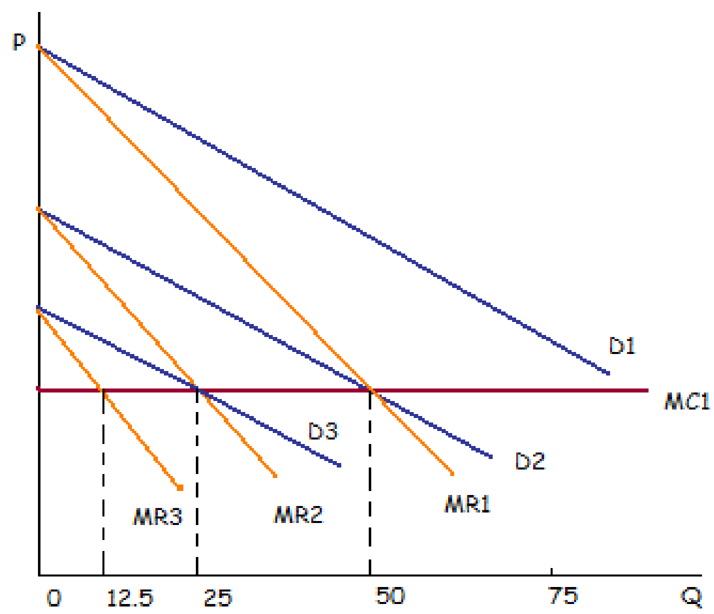

FIGURE 3.2: FIRM A's OUTPUT DECISION

Thus, firm A's profit maximizing output is a decreasing schedule of how much it thinks firm B will produce. This schedule is firm A's reaction curve denoted by $Q_A{}^*Q_B$ in Figure 3.3.

The same can be shown for firm B; that is, we can determine firm B's profit maximizing output given various assumptions about how much firm A will produce. This gives the reaction curve for firm B, denoted by $Q_B{}^*Q_A$.

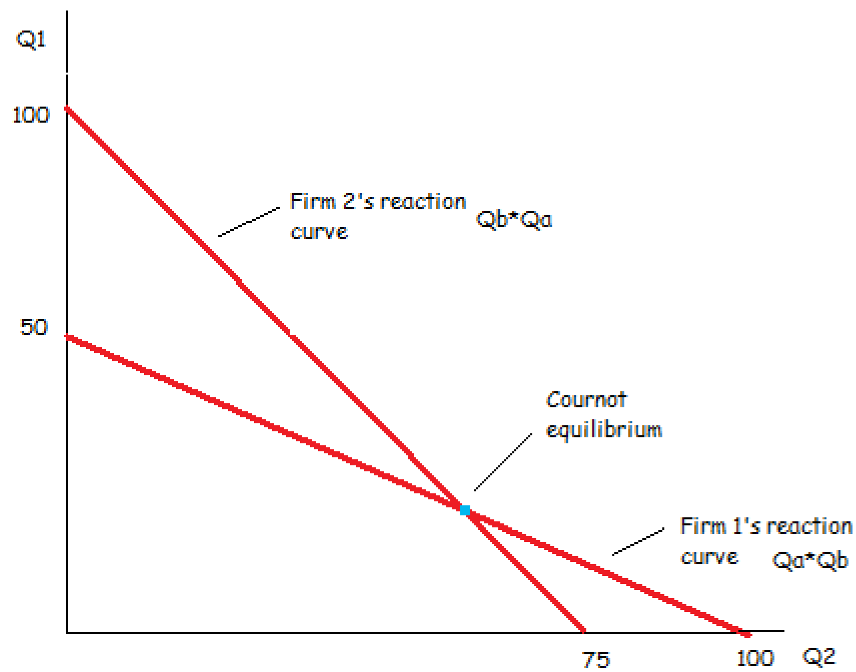

FIGURE 3.3: REACTION CURVES AND COURNOT EQUILIBRIUM

Each firm's reaction curve tells it how much to produce, given the output of its competitor. In equilibrium, each firm sets output according to its own reaction curve; the equilibrium output levels are therefore found at the intersection of the two reaction curves. This resulting set of output levels is called the **Cournot equilibrium**. In this equilibrium, each firm correctly assumes how much its competitor will produce, and it maximizes its profit accordingly.

4. Bertrand's Duopoly Model

In the Cournot model, it is assumed that the oligopolistic firms compete by setting quantities. In many oligopolistic industries, however, competition occurs along price dimensions. For example, automobile companies view price as a key strategic variable, and each one chooses its price with its competitors in mind. Price competition in an oligopoly industry can be studied with the help of Bertrand model.

The **Bertrand model** was developed by Joseph Bertrand. Like the Cournot model, it applies to firms that produce the same homogenous good and make their decisions simultaneously. In this case, however, the firms choose prices instead of quantities.

Thus each firm is faced by the same market demand, and aims at the maximization of its own profit on the assumption that the price of the competitor will remain constant. The model can be presented using reaction functions of the duopolists. In Bertrand's model, the reaction curves are derived from isoprofit maps which are convex to the axes measuring the prices of the duopolists. Each isoprofit curve for firm A shows the same level of profit which would accrue to A from various levels of prices charged by this firm and its rival. The isoprofit curve for A is convex to its price axis (P_A). This shape implies that firm A must lower its price up to a certain level (point *e* in Figure 4.1) to meet the cutting of price of its competitor, in order to maintain the level of its profits at π_{A2}. However, after that price level has been reached and if B continues to cut its price, firm A will be unable to retain its profits, even if it keeps its own price unchanged (at P_{Ae}). If firm B cuts its price at P_B, firm A will find itself at a lower isoprofit curve (π_{A1}) which shows lower profits. Clearly, the lower the isoprofit curve, the lower the level of profits.

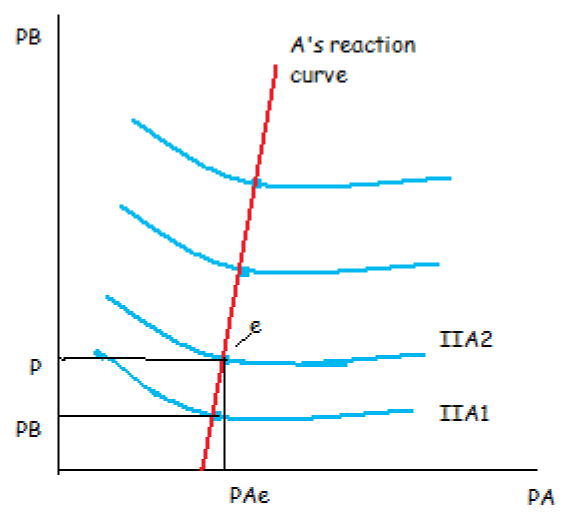

FIGURE 4.1: FIRM A's REACTION CURVE

Thus for any price charged by firm B there will be a unique price of firm A which maximizes the latter's profit. This unique profit-maximizing price is determined at the lowest point on the highest attainable isoprofit curve of A. The minimum points of the isoprofit curves lie to the right of each other, reflecting the fact that as firm A moves to a higher level of profit, it gains some of the customers of B when the latter increases its price, even if A also raises its price. On joining the lowest points of the successive isoprofit curves, we obtain the reaction curve of firm A. This reaction curve shows the locus of points of maximum profits that A can attain by charging a certain price, given the price of its rival.

The reaction curve of firm B can be derived in a similar way, by joining the lowest points of its isoprofits curves (Figure 4.2).

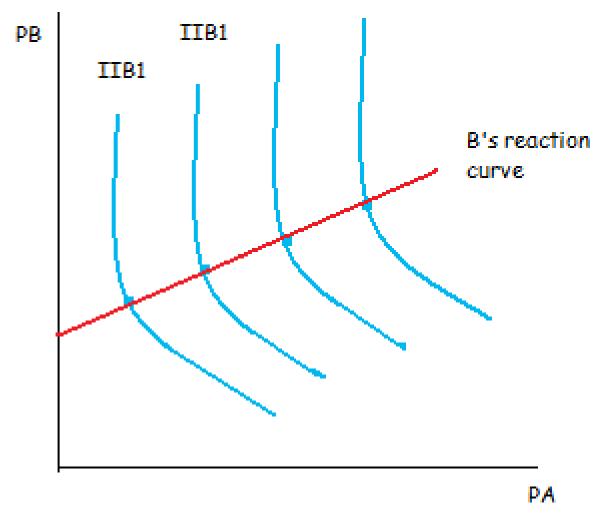

FIGURE 4.2: FIRM B's REACTION CURVE

Bertrand's model leads to a stable equilibrium, defined by the point of intersection of the two reaction curves (Figure 4.3).

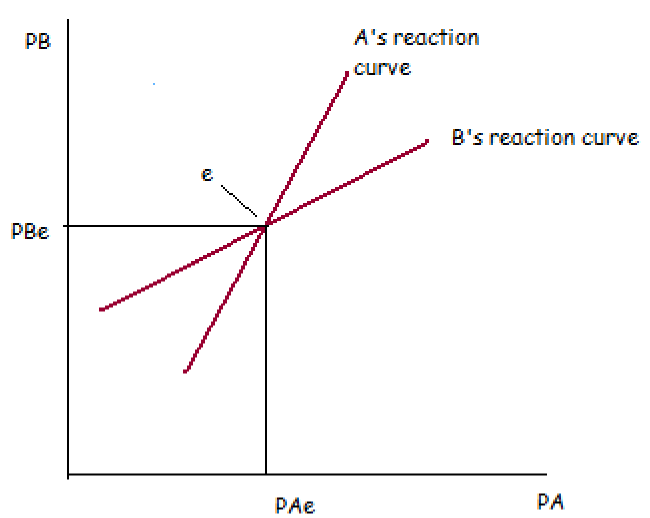

FIGURE 4.3: BERTRAND EQUILIBRIUM

Point *e* denotes a stable equilibrium, since any departure from it sets in motion forces which will lead back to point *e* at which the price charged by A and B are P_{Ae} and P_{Be} respectively.

5. Stackelberg's Duopoly Model

This model was developed by Heinrich von Stackelberg and is an extension of Cournot's model. It is assumed that one duopolist is sufficiently sophisticated to recognize that his competitor acts on the Cournot assumption. This allows the duopolist to determine the reaction curve of his rival and incorporate it in his own profit function, which he then proceeds to maximize like a monopolist.

Consider Figure 5.1 which shows the isoprofit curves and reaction functions of the duopolists (similar to the Cournot model). The reaction curve of firm A is the locus of points of highest profits that it can attain, given the level of output of rival B. It shows how firm A will determine its output as a reaction to B's decision to produce a certain level of output. Similarly, each point on B's reaction curve shows how much output B must produce in order to maximize its own profit, given the level of output of its rival firm A.

If firm A is the sophisticated duopolist, it will assume that its rival will act on the basis of its own reaction curve. This permits firm A to choose to set its own output at the level which maximizes its own profit. This is the point *a* (in Figure 5.1) which lies on the lowest possible isoprofit curve of A, denoting the maximum profit A can achieve given B's reaction curve. Firm A, acting as a monopolist (by incorporating B's reaction curve in his profit-maximizing calculations) will produce X_A, and firm B will react by producing X_B according to its reaction curve. The sophisticated duopolist becomes the leader, while the rival firm which acts on the Cournot assumption becomes the follower. Being a leader is rewarding for A because he reaches an isoprofit curve closer to his axis than if he behaved like his rival. The follower is worse off as compared with the Cournot equilibrium, since with this level of output he reaches an isoprofit curve further away from his axis.

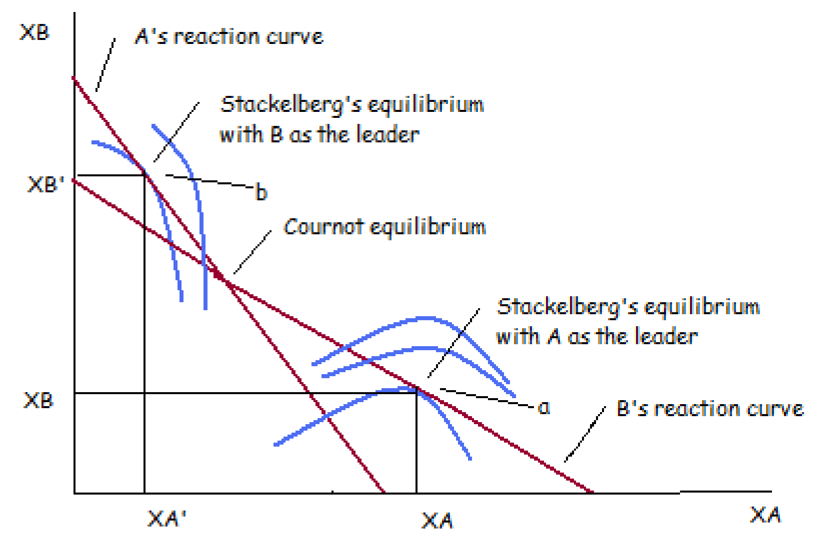

FIGURE 5.1: STACKELBERG EQUILIBRIUM

If B is the sophisticated duopolist, it will choose to produce X_B, corresponding to point *b* on A's reaction curve because this is the largest profit that B can achieve given his isoprofit map and A's reaction curve. Firm B will now be the leader while firm A becomes the follower. B has a higher profit and firm A has a lower profit as compared with the Cournot equilibrium.

Thus, if only one firm is sophisticated, it will emerge as the leader, and a stable equilibrium will emerge, since the rival firm will act as a follower.

However, if both firms are sophisticated, then both will want to act as leaders, because this action yields a greater profit to them. In this case, the market situation becomes unstable. The situation is known as Stackelberg's disequilibrium and the effect will either be a price war until one of the firms surrenders and agrees to act as follower, or a collusion is reached, with both firms attaining higher profits.

The Stackelberg model has interesting implications.

- It shows that the rivals should recognize their interdependence. By recognizing the other's reactions each duopolist can reach a higher level of profit for himself.
- If both firms start recognizing their mutual interdependence, each starts worrying about the rival's profits and the rival's reactions. If each ignores the other, a price war will be inevitable, as a result of which both will be worse off.
- This model of sophisticated behavior is not applicable in a market in which the firms behave on Bertrand's assumption. In a Bertrand-type market the sophisticated duopolist can do nothing which would increase his own profit and persuade the other to stop price-cutting. The most he can do is to keep his own price constant, that is, behave exactly as his opponent expects him to behave.

6. Numerical Illustration of Cournot and Stackelberg models

Assume that the market demand and the costs of duopolists are

$$P = 100 - 0.5(X_1+X_2)$$
$$C_1 = 5X_1$$
$$C_2 = 0.5X_2^2$$

The profits of the duopolists are

(a) $\Pi_1 = PX_1 - C_1 = [100 - 0.5(X_1+X_2)]\, X_1 - 5X_1$

Or

$$\Pi_1 = 100X_1 - 0.5X_1^2 - 0.5X_1X_2 - 5X_1$$

(b) $\Pi_2 = PX_2 - C_2 = [100 - 0.5(X_1+X_2)]\, X_2 - 0.5X_2^2$

Or

$$\Pi_2 = 100X_2 - 0.5X_2^2 - 0.5X_1X_2 - 0.5X_2^2$$

Collecting terms we have

$$\Pi_1 = 95X_1 - 0.5X_1^2 - 0.5X_1X_2 \qquad \ldots\ldots(1)$$

And

$$\Pi_2 = 100X_2 - X_2^2 - 0.5X_1X_2 \qquad \ldots\ldots.(2)$$

For profit-maximization under the Cournot assumption we have

$$\partial\Pi_1/\partial X_1 = 0 = 95 - X_1 - 0.5X_2$$

$$\partial\Pi_2/\partial X_2 = 0 = 100 - 2X_2 - 0.5X_1$$

The reaction functions are:

$$X_1 = 95 - 0.5X_2 \qquad \ldots\ldots(3)$$
$$X_2 = 50 - 0.25X_1 \qquad \ldots\ldots(4)$$

Cournot equilibrium is the point of intersection between the two reaction curves.
This implies substituting (4) in (3), we get

$$X_1 = 95 - 0.5(50 - 0.25X_1)$$

Or

$$X_1 = 80$$

And

$$X_2 = 50 - 0.25X_1 = 50 - (0.25)(80) = 30$$

Thus the total output in the market is

$$X = X_1 + X_2 = 110$$

And the market price

$P = 100 - 0.5X = 45$

The profits of the duopolists are

$\Pi_1 = PX_1 - C_1 = (45)(80) - 5(80) = 3200$

And

$\Pi_2 = PX_2 - C_2 = (45)(30) - 0.5(30)^2 = 900$

Cournot solution is:

$X_1 = 80$

$X_2 = 30$

$X = 110$

$P = 45$

$\Pi_1 = 3200$

$\Pi_2 = 900$

Stackelberg's solution has three situations as follows:

(a) *Stackelberg's solution with A being the sophisticated leader*

In this case firm A will substitute B's reaction function [computed as (4)] in its own profit function [given by (1)], and maximize it—

$\Pi_1 = PX_1 - C_1 = 95X_1 - 0.5X_1^2 - 0.5X_1X_2$

Substitute $X_2 = 50 - 0.25X_1$

Maximize $\Pi_1 = 70X_1 - 0.375X_1^2$

First-order condition: $\partial\Pi_1/\partial X_1 = 0 = 70 - 0.75X_1$

This yields output: $X_1 = 93\frac{1}{3}$

And profit: $\Pi_1 = 70X_1 - 0.375X_1^2 = 3267$

The second-order condition for profit maximization is fulfilled.

Firm B would be the follower. It would assume that A would produce 93⅓ units; thus B substitutes this amount in its reaction function

$X_2 = 50 - 0.25X_1 = 26\frac{2}{3}$

And its profit would be

$\Pi_2 = 100X_2 - X_2^2 - 0.5X_1X_2 = 711.1$

(b) *Stackelberg's solution with firm B being the sophisticated leader*

In this case, Firm B will substitute A's reaction function [computed as (3)] in its own profit function [given as (2)], and maximizes this profit as a monopolist—

$$\Pi_2 = PX_2 - C_2 = 100X_2 - X_2^2 - 0.5X_1X_2$$

By substituting, $X_1 = 95 - 0.5X_2$, we get

$$\Pi_2 = 52.5X_2 - 0.75X_2^2$$

The first-order profit-maximizing condition is:

$$\partial\Pi_2/\partial X_2 = 0 = 52.5 - 1.5X_2$$

Which yields output: $X_2 = 35$

And profit: $\Pi_2 = 52.5X_2 - 0.75X_2^2 = 918.75$

The second-order condition for profit maximization is fulfilled.
The follower is now firm A which assumes that the rival will keep his quantity at $X_2 = 35$ units, and calculates its own output by substituting this quantity in its reaction function.

$$X_1 = 95 - 0.5X_2 = 77.5$$

And its profit is $\Pi_1 = 95X_1 - 0.5X_1^2 - 0.5X_1X_2 = 3003.13$

(c) *Stackelberg's disequilibrium*

If both firms adopt Stackelberg's sophisticated pattern of behavior, each will examine his profits if he acts as a leader and if he acts as a follower, and will adopt the action that will yield him the greatest profit.

Firm A calculates its profits both as a leader and as a follower:
If A is the leader his profits are 3267
If A is the follower his profits are 3003.13

Clearly firm A will prefer to act as the leader.

Similarly, Firm B calculates its profits both as a leader and as a follower:
If B is the leader his profits are 918.75
If B is the follower his profits are 711.1

Thus firm B will also choose to act as the leader.
With both firms acting in the sophisticated way implied by Stackelberg's behavioural hypothesis both will want to act as leaders. As they attempt to do so they find that their expectations about

the rival are not fulfilled and warfare will start, unless they decide to come to a collusive agreement.

7. Summary

- Oligopoly is a market structure where only a few firms account for most or all of production.
- Barriers to entry allow some firms to earn substantial profits.
- Economic decisions involve strategic considerations --- each firm must consider how its actions will affect its rivals, and how they are likely to react.
- In the Cournot model, firms make their output decisions at the same time, each taking the other's output as fixed.
- In equilibrium, each firm is maximizing its profit, given the output of its competitor, so no firm has an incentive to change its output.
- In the Stackelberg's model, one firm sets its output first. That firm has a strategic advantage and earns a higher profit.
- The Bertrand model applies to markets in which firms compete by setting price. In equilibrium, each firm maximizes its profit, given the prices of its competitors, and so has no incentive to change price.