Paper 4: Operations Management

z **Module 32: Inventory Models: Instantaneous Replenishment**

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Paper 4

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Module 35: Inventory Models: Instantaneous Replenishment

32.0 OBJECTIVES

This chapter would help students to understand:

- Application of Instantaneous Replenishment model
- Assumptions of Economic Order Quantity (EOQ) model
- Calculation of optimum batch size and minimum total inventory cost

32.1 INTRODUCTION

Inventory is the stock or resource that a firm stores for carrying out operations. Every firm whether in service or manufacturing sector faces with two primary issues related with inventory: how much inventory to keep and when to replenish the stock. These two questions have been quite accurately answered even before the implementation of new software and technologies. The objective of every firm is to keep inventory cost at minimum and to do so the above mentioned two questions have to be answered as accurately as possible. The accuracy of answers depend on preciseness of data collected related to number of items used, forecast of each item, variation in demand, resources available to manufacture items etc. With decreasing product life cycles and increasing competition variation and amount of such data is huge which makes necessary to use sophisticated techniques for solution of inventory problems. Inventory models deals with such problems and provide answers to questions of how much and when to order in such a manner so that total cost is most optimum. There are different models available depending on assumptions applied for production and type of sector. Following illustration indicate the importance of application of these models in both service and manufacturing sector:

- An auto ancillary unit is manufacturing emblems and wheel covers of various car companies. The unit has to take into account type of emblems and amount of each type required by a particular car company. For instance, Maruti Suzuki uses insignia of Suzuki as well name of car such as Dezire etc. as emblems. Now insignia is a constant for all models but name of car changes with each type of car. Auto ancillary unit faces with a problem of how much of raw material to be ordered to manufacture these two parts. Problem becomes more severe when demand of other car companies is also added. Inventory models help to resolve such issues.
- A retailer who stores variety of products has to constantly be cautious of customer demand for different kind of products. Storage of one type of product requires space and maintenance. If its demand is low then it would result in wasteful expenditure whereas retailer would lose customers for items which are in demand. Understanding of application of different inventory models would help a retailer to reduce its inventory cost and fulfill customers' demand.

32.2 INSTANTANEOUS REPLENISHMENT MODEL: ECONOMIC ORDER QUANTITY (EOQ)

In resolving the conflicting issue of keeping low inventories to reduce holding costs but high enough to reduce frequency of orders understanding of EOQ is important. This model as the title suggests calculates size of order that should be placed with supplier which would keep both cost of carrying it and placing new orders at most optimum. Following terms should be understood before evaluating EOQ model:

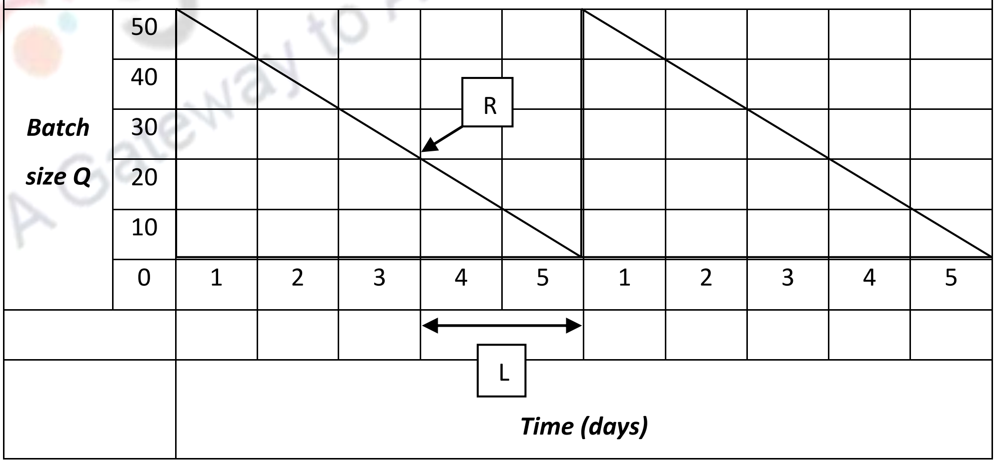
- ***Holding cost (H):*** also termed as carrying cost is the cost associated with holding inventory in hand. They include storage costs, spoilage costs, obsolescence costs etc. For instance a warehouse storing grains has to spend money on its maintenance and prevent it from getting spoiled. Thus such warehouse incurs much of its expenditure in holding grains.
- ***Ordering cost (S):*** also termed as set-up cost are incurred on obtaining additional inventories. They include costs of communicating new order, transportation costs etc. In manufacturing when machine has to stop for replacement of tools to produce different item then time is lost in production because of setting up of process. That is why this cost is also called as set-up cost.
- ***Reorder point (R):*** is the time when new order is placed. This also indicates the point when out of total stock how much is consumed and how much is left. Estimation of reorder point is crucial for unhindered production as when new order is placed then manager should be sure of having that much stock which can be used till stock is replenished with new order.
- ***Lead time (L):*** is the time taken by the supplier to replenish the stock. Variation in lead time can play havoc with inventory management. Delay in replenishment can lead to stock out and loss of production or customers whereas making available stock before time can lead to excess stock.

32.2.1 Assumptions of EOQ

EOQ model can work accurately only if it follows following assumptions:

- ***Demand rate is constant:*** the inclined line in the figure (Fig. 1) indicates that daily demand out of stock of 50 units is constant of 10 units. Thus units are consumed at fixed, constant rate.
- ***Size of each lot is constant:*** in every ordering cycle number of units ordered is same. It also implies that there are no constraints in production. A supplier can always without any bottlenecks would be able to provide a fixed batch size of Q.
- The model takes into consideration only ***holding and ordering costs***.
- ***Reorder point is constant:*** Every order in each ordering cycle is placed at same time or when same number of units is left in the stock. For instance, a batch size of 50 units, daily demand of 10 units and lead time of 2 days implies that next order should be placed when 2*10=20 units are left in the stock. So, in case of every ordering cycle next order would be place after 3 days or when 20 units are left in the stock as shown in Fig. 1.
- ***Lead time is fixed and constant.*** Supplier would provide a fixed batch size in fixed and certain time period.
- Model should be applied only for those items whose ***demand is independent*** of other items.
- One of the most important assumptions of EOQ model is that stock gets replaced ***instantaneously*** that is why the model is also called as instantaneous replenishment model. As the figure shows on X axis is the time and Y axis represents a batch size. At time t=0 stock is full with batch size of Q=50m units. With each passing day the batch is consumed at constant rate of 10 units and after time t=5 days entire batch gets consumed i.e Q=0. But at the same point in time stock gets replenished with same batch size of Q=50 units (Fig. 1). This makes EOQ model as instantaneous replenishment model.

Fig. 1: EOQ model



32.2.2 Calculation of EOQ

With the understanding of above mentioned assumptions of EOQ following procedure is adopted for formulating the quantitative formula for calculating number of units to be ordered per batch and total cost. As discussed total cost would be a function of only ordering and holding cost. Notations used in the model are:

D: Annual demand

H: Holding cost per unit

S: Ordering or set-up cost per order

Q: Batch size

Holding cost: Between time period of start and end of cycle (time period t) on an average at any particular unit Q/2 units are being held up in inventory. As Fig. 2 shows with increase in number of units stored in inventory annual holding cost increases. If to hold one unit holding cost is H then:

Annual holding cost = Average inventory level * Holding cost per unit

= (Q/2) * H

Ordering cost: Suppose annual demand of an item is 1000 units and manager orders 100 units per order. Thus he/she has to place an order 10 times. Thus (D/Q) represents number of orders. As Fig. 2 shows with increase in number of units stored in inventory annual ordering cost decreases. If S is set up or ordering cost per order then:

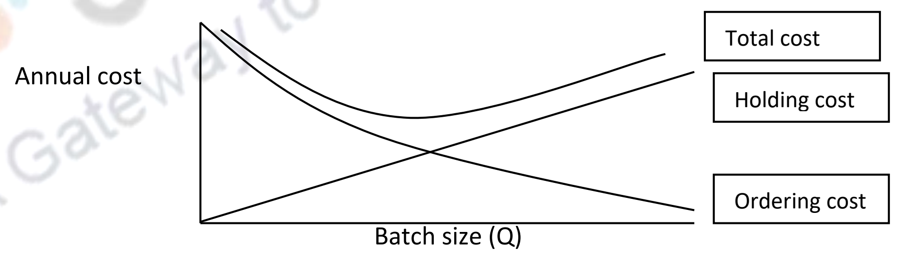
Annual ordering cost = Number of orders per year * Ordering or set up cost per order

= (D/Q) * S

Figure 2 deduces a very important relationship between holding and ordering cost. The graph clearly shows that as number of units stored increase holding cost keeps on increasing and ordering cost decreases. This is quite understandable as with bigger batch size frequency of repeating orders automatically decreases. ***Thus ordering and holding cost have an inverse relationship with each other.***

Importantly, the intersection of holding and ordering cost curve indicates number of units that should be ordered per batch. At this intersection point the most optimum total annual cost can be computed.

Fig. 2: Cost curves in EOQ



As total cost is a combination of two types of cost so:

Total cost = Annual holding cost + Annual ordering cost

TC = (Q/2) * H + (D/Q) * S

To find minimum possible cost the above equation is differentiated w.r.t. Q and simple minima calculus is applied to find out batch size Q which would be

Q = √(2DS/H)

This formula of Q would provide manager the most optimum batch size that should be ordered with minimum total annual cost.

32.3 PRACTICAL EXAMPLES

Example 1: Yellow press buys paper in 1500 pound rolls for textbook printing. Annual demand is 2500 rolls. The cost per roll is Rs.800 and annual holding cost is 15% of the cost. Each order costs Rs.50. How many rolls should Yellow press order at a time? What would be the total cost of keeping that many rolls?

Solution: Annual demand D = 2500

Holding cost, H = 0.15 * 800 = Rs.12

Ordering cost, S = Rs. 50

Number of rolls to be ordered:

$$Q = \sqrt{(2DS/H)}$$

$$Q = \sqrt{(2*2500*50/12)}$$

$$= 144.3 \text{ units} = \mathbf{144 \text{ units}}$$

Total annual cost would be:

$$TC = (Q/2) * H + (D/Q) * S$$

$$= (144/2) * 12 + (2500/144) * 50$$

$$= 864 + 868.055$$

$$= \mathbf{1732.05}$$

Example 2: A firm buys 400 units of tapes per month for use in production content. The ordering cost is Rs.12.50. Holding cost is Rs.0.12 per tape. How many tapes should company order at a time? What is the time between orders?

Solution: Annual demand D = 400*12 = 4800

Ordering cost S = Rs. 12.50

Holding cost H = Rs.0.12

Thus, $Q = \sqrt{(2*4800*12.50/0.12)}$

$= 1000$ units

Another concept that is discussed in this example is of ***Time between orders (TBO)***. It represents the average time elapsed between receiving replenishment of Q units. Thus id D i.e. annual demand is consumed in 12 months or 52 weeks or 365 days then Q units would be consumed in (Q/D) time period.

So **TBO = Q/D**

Thus TBO = 1000 / 4800

= 0.208

In months TBO = 0.208 * 12

= 2.5 months

Thus this implies that a batch size of 1000 units is consumed in 2.5 months and stock is replenished after every 2.5 months.

Example 3: Demand per week is 18 units. Price per unit is Rs.60. Holding cost = 25% of price of product. Cost of placing an order = Rs.45.

(a) How many units should be ordered per batch?

(b) If in addition to batch size as computed in (a) is 390 units and another batch is 468 units then which batch should be ordered? Also calculate TBO for each batch size.

Solution:

(a) D = 18 * 52 = 936 units

H = 0.25*60 = Rs.15

S = Rs. 45

Batch size	**Q**	**=**	**√(2DS/H)**
	Q	=	√(2*936*45/15)
		=	75 units

(b) To decide which batch size is most optimum find Total cost for each batch size i.e.

Total cost (when Q = 75 units)	=	(Q/2) * H	+	(D/Q) * S
	=	(75/2) * 15	+	(936/75) * 45
	=	562.5	+	561.6
	=	**1124.1**		
TBO	**=**	**Q/D**		
	=	75/18		
	=	4.16 weeks		
Total cost (when Q = 390 units)	=	(Q/2) * H	+	(D/Q) * S
	=	(390/2) * 15	+	(936/390) * 45
	=	**3033**		
TBO	**=**	**Q/D**		
	=	390/18		
	=	21.66 weeks		
Total cost (when Q = 468 units)	=	(Q/2) * H	+	(D/Q) * S
	=	(468/2) * 15	+	(936/468) * 45
	=	**3600**		
TBO	**=**	**Q/D**		
	=	468/18		
	=	26 weeks		

This implies that with increase in batch size total annual cost keeps on increasing, so it would be preferable to have a batch size of 75 units. Also with increase in batch size TBO keeps on decreasing.

32.4 AN EXAMPLE OF LEAN MANUFACTURING TECHNIQUE IN REDUCING INVENTORY

Inventory means having unnecessarily high levels of raw materials, works-in-process and finished products. Extra inventory leads to higher inventory financing costs, higher storage costs and higher defect rates. It tends to increase lead time, prevents rapid identification of problems and increase space requirements. In order to conduct effective purchasing, it is especially necessary to eliminate inventory due to incorrect lead times.

Work in Progress (WIP) is a direct result of overproduction. Excess inventory tends to hide problems on the plant floor, which must be identified and resolved in order to improve operating performance. Excess inventory increases lead times, consumes productive floor space, delays the identification of problems, and inhibits communication. By achieving a seamless flow between work centers, many manufacturers have been able to improve customer service and slash inventories and their associated costs.

The ***Just-in-Time (JIT)*** philosophy advocates Producing and/or delivering only the necessary parts, within the necessary time in the necessary quantity using the minimum necessary resources. Ideally, the appropriate number of parts are produced and immediately shipped when the customer order is received. Upstream processes and suppliers deliver exactly the appropriate quantity of components when the downstream process needs them. In this situation there is no need for inventory. Eliminating all inventory and work-in-process (WIP) is impossible in the practical sense. The key to manufacturing efficiency is continuously decreasing the quantity of each in the system. There is a general tendency to react to problems by accumulating a reserve of stock based on an estimate of quality defects, equipment breakdown and team member absenteeism.

Single-Minute Exchange of Die (SMED):

SMED provides a rapid and efficient way of converting a manufacturing process from running the current product to running the next product. This rapid changeover is key to reducing production lot sizes and thereby improving flow, reducing production loss and output variability. The phrase "single minute" does not mean that all changeovers and start-ups should take only one minute, but that they should take less than 10 minutes (in other words, "single-digit minute"). Closely associated is a yet more difficult concept, One-Touch Exchange of Die, (OTED), which says changeovers can and should take less than 100 seconds. A die is a tool used in manufacturing. However SMED's utility of is not limited to manufacturing. There are seven basic steps to reduce changeover using the SMED system:

- *Observe* the current methodology.
- *Separate the Internal and External activities*. Internal activities are those that can only be performed when the process is stopped, while External activities can be done while the last batch is being produced, or once the next batch has started. For example, go and get the required tools for the job before the machine stops.
- *Convert* (where possible) Internal activities into External ones (Pre-heating of tools is a good example of this).
- *Streamline* the remaining internal activities, by simplifying them.
- *Streamline* the External activities, so that they are of a similar scale to the internal ones.
- *Document* the new procedure, and actions that are yet to be completed.
- *Do it all again*. For each iteration of the above process, a 45% improvement in set-up times should be expected, so it may take several iterations to cross the ten-minute line.

32.5 SUMMARY

Economic Order Quantity (EOQ) is an inventory model which is used to evaluate quantity per batch to be ordered so that total cost is kept at minimum. In this model total cost is a function of only holding and ordering cost. EOQ model works under certain assumptions. Some of these assumptions happen only under ideal conditions making scope of this model under very limited conditions. One of the most important assumptions of replenishment of stock at the same time when stock gets finished makes EOQ as instantaneous replenishment model.

32.6 GLOSSARY

- ***Economic order quantity***: is an inventory model that determines quantity to be ordered per batch so that total cost is minimum.
- ***Holding cost:*** is cost incurred for storing items as stock.
- ***Ordering cost:*** is cost incurred in placing or communicating orders.
- ***Time between orders:*** represents the time taken for consumption of number of units per batch.

32.7 REFERENCES/ SUGGESTED READINGS

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- Lee J. Krajewski, *Operations Management,* Prentice-Hall of India, New Delhi, 8th Edition.

32.8 SHORT ANSWER QUESTIONS

1. EOQ is the order quantity that over our planning horizon.

a) minimizes total ordering costs b) minimizes total carrying costs

c) minimizes total inventory costs d) the required safety stock

Answer: c

2. Suppose that the EOQ for an inventory problem was first calculated to be Q*. Then you found out that the ordering cost would be lower than anticipated. Without recalculating the EOQ, what can you say about the relationship between the new Q* (= Q*new) and the original Q*?

a) Q* = Q*new

b) b. Q* < Q*new

c) c. Q* > Q*new

d) Inconclusive without knowing how much the ordering cost decreased

Answer: b

3. Inventory holding cost may include

a) material purchase cost

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b) interest on loans

c) penalty charge if demand is not met

d) inspection of incoming supplies at warehouses

Answer: b

32.9 MODEL QUESTIONS

1 The University Gift Shop purchases sweatshirts emblazoned with the school name and logo from a vendor in Toronto. The vendor sells the sweatshirts to the Gift Shop for $34.99 apiece. Shipping from Toronto to London costs $110 per order. When an order arrives, it has been estimated that receiving and inspection tasks cost the Gift Shop $25. The annual holding cost for a sweatshirt is calculated as 11% of the purchase cost. The Gift Shop manager estimates that 3100 sweatshirts will be sold during the upcoming academic year. Determine the optimal order quantity using the basic EOQ model.

2 What is the relationship between holding and carrying cost? Explain.

3 How batch size and total annual cost of inventory is evaluated?

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