Subject	COMMERCE
Paper No and Title	14. Security Analysis and Portfolio Management
Module No and Title	M31. Capital Asset Pricing Model
Module Tag	COM_P14_M31

Principal Investigator	Co- Principal Investigator		Co- Principal Investigator and Technical Coordinator
Prof K V Bhanu Murthy Professor Department of Commerce University of Delhi	**Dr. Jaswinder Singh** Principal SGTB Khalsa College University of Delhi	**Dr. R P Singh** Associate Professor SGTB Khalsa College University of Delhi	**Dr Vimal Rarh** Deputy Director, Centre for e-Learning and Assistant Professor, Department of Chemistry, SGTB Khalsa College, University of Delhi *Specialised in : e-Learning and Educational Technologies*
Paper Coordinator	**Content Writer**		**Reviewer**
Prof. H.K. Singh Professor, Faculty Of Commerce, BHU, Varanasi.	**Prof. D. Sahu** Professor, Faculty Of Commerce, BHU, **Ms. Rupali Gupta** Research Scholar, Faculty of Commerce, BHU, Varanasi		**Prof. H.K. Singh** Professor, Faculty Of Commerce, BHU, Varanasi
Anchor Institute : SGTB Khalsa College, University of Delhi			

TABLE OF CONTENTS

4. Riskless Lending and Borrowing

5. Capital Asset Pricing Model
5.1 Assumptions

6. Capital Market Line

7. Security Market Line

8. Summary

1. Learning Outcomes

After studying this module, you shall be able to

- Understand the concept of Capital Asset Pricing Model
- Analyze the Capital market Theory and CAPM
- Learn the concept of risk free asset, risk free lending and borrowing
- Discuss the assumptions of model
- Explain the concept of Capital market Line and Security Market Line
- Create efficient frontier under CAM and CML

2. Introduction

The Markowitz Portfolio Theory explains how the investors should choose the efficient portfolio based upon their utility preferences. The Capital Market Theory is built upon and is an extension of the Markowitz Model. In that it hypotheses how the investors do behave rather than how the investors should behave as explained by the Markowitz Model. The Capital Market Theory develops a model for determining the price of risky assets in the capital market. Risky assets are those assets that carry an uncertain amount or rate of return. In fact, these depend upon the market forces and the efficiencies in the market system. For example- equity shares, corporate debentures, mutual funds, etc. the concept of risk free assets is very vital to the development of capital market theory. Risk free assets are those assets or investment avenues that carry a fixed or certain amount or percentage of return. These are free from default risk. But in practice, these are still prone to risks arising from fluctuations in the interest rates and inflation rates, as a result of which the real returns get negatively affected. Examples of risk free assets are- treasury bills, dated government securities, etc.

3. Assumptions of Capital Market Theory

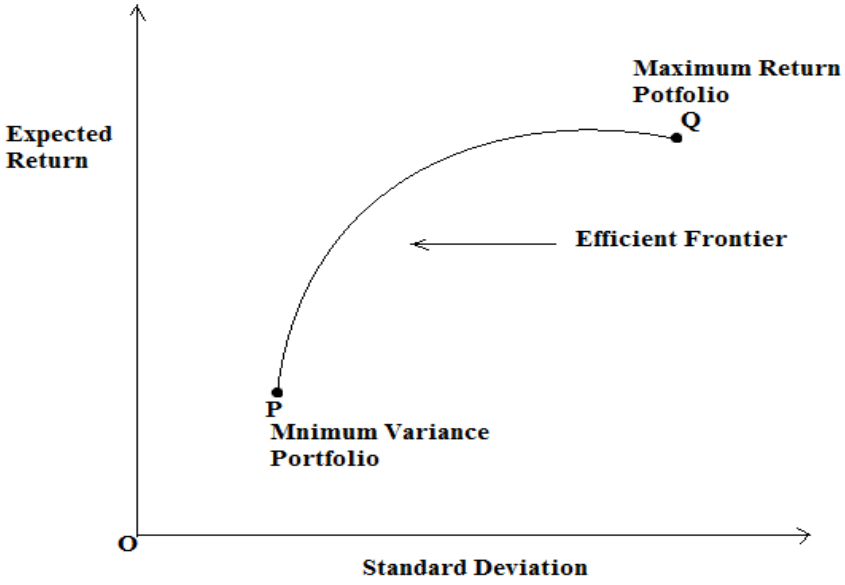
The main assumptions of CMT are as follows:

- All investors are expected to make decisions based solely on their assessment of risk-return. They follow the idea of Markowitz Efficient Frontier and choose to invest along the efficient frontier.
- The investors are assumed to lend and borrow any amount of money at the risk- free rate.
- The time horizon for choosing investments is equal for all investors.
- The purchase and sale transactions can be taken in infinitely divisible units.
- The personal income taxation is assumed to be zero and there exists no transaction cost i.e. there is perfect competition in the market.
- Returns are not affected by the inflation rates in a capital market as none exists in the capital market theory.
- It is assumed that the markets are efficient and there is no mispricing of assets within the market.

4. Riskless Lending and Borrowing

The capital market theory assumes that there exists riskless lending and borrowing among the investors. In other words, investors can lend/borrow any amount of money at the risk free rate (R_f). As explained by the Markowitz Portfolio theory, the efficient frontier is the concave part of the curve that extends from the minimum variance (risk) portfolio to the maximum return portfolio. The efficient frontier assumes that all the securities on the efficient set are risky. This is shown in Figure .1.

Figure .1



Efficient Frontier of Risky Assets

With the introduction of risk free assets, this efficient frontier undergoes a change. The correlation between all the points in the feasible portion of risky portfolios and the risk free rate (R_f) is zero (0).

Suppose that an investor is interested to place a portion of his investments in the risky portfolio A (a point on the efficient frontier) and there remaining portion in the risk free asset. This combination is a new portfolio. Let us denote *x* as the portion of investment in A and *(1-x)* is the portion of risk free asset.

Now, the expected return and the variance of the new portfolio is given as under:

$E(R_P) = [x\ E(R_A) + (1-x)R_f]$

.....Equation.1

And,

$\sigma_P^2 = x^2 * \sigma_A^2 + (1-x)^2\sigma_f^2 + 2x(1-x)\sigma_{Af}$

.....Equation.2

Since, σ_f = 0 and σ_{Af} = 0, then variance is simply given by

$\sigma_P^2 = x^2\ \sigma_A^2$

$\sigma_P = \sqrt{x^2\sigma_A^2} = x\,\sigma_A$
Therefore, $x = {}^{\sigma_P}/_{\sigma_A}$

.....Equation.3

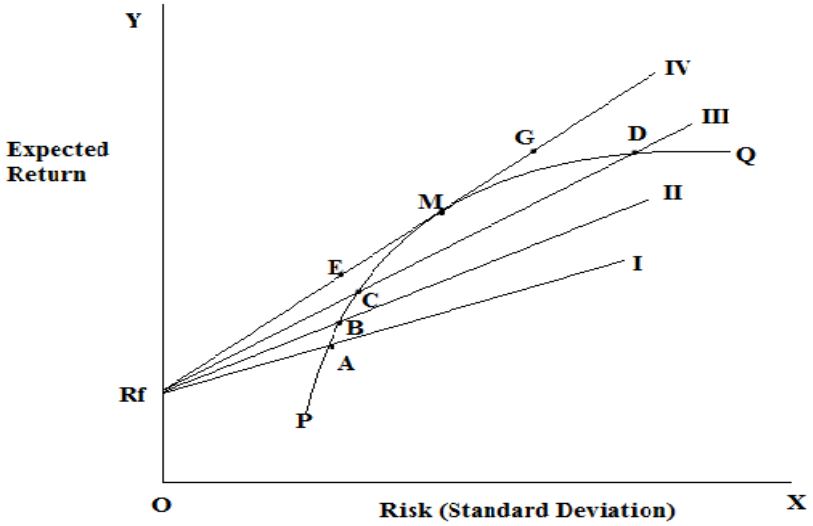
By substituting equation 36.3 in equation 36.1, we get,
$E(R_P) = {}^{\sigma_P}/_{\sigma_A} * E(R_A) + (1 - {}^{\sigma_P}/_{\sigma_A}) * R_f$
$= {}^{\sigma_P}/_{\sigma_A} * E(R_A) + R_f - {}^{\sigma_P}/_{\sigma_A} * R_f$
$= R_f + {}^{\sigma_P}/_{\sigma_A}\{E(R_A) - R_f\}$

This implies; $E(R_P) = R_f + \{\frac{E(R_A) - R_f}{\sigma_A}\}\,\sigma_P$

.....Equation.4

Graphically, equation .4 is a straight line that has an intercept of R_f and slope $\{\frac{E(R_A) - R_f}{\sigma_A}\}$ and it passes through portfolio A. Portfolio A was chosen arbitrarily. Likewise some other investors may choose portfolio B, portfolio C or portfolio D. What would be the best combination for the investor?
Clearly a line that is tangent to the efficient frontier would provide a solution to this problem. This tangent line dominates any other line between R_f and any point in the feasible region of risky securities.
This is shown in Figure .2

Figure .2



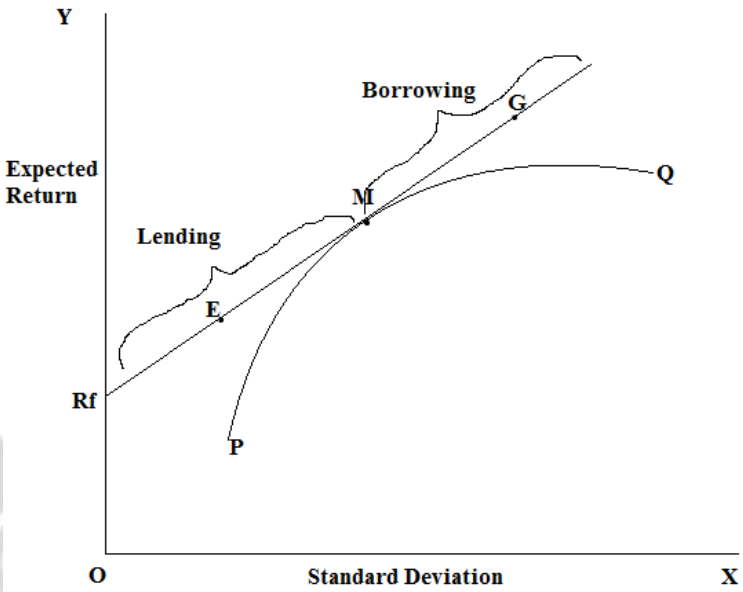
Efficient Frontier with Risk Free Rate

Figure 36.2 shows the efficient frontier with lending and borrowing opportunities. Although the investor can choose either of the portfolios A, B, C or D along the efficient frontier curve PCQ or reach any point on lines I, II, or III; none of the point is optimal. Line IV is tangent to efficient set

of risky securities at pint M. it provides the best possible opportunities to the investors. It is clear from the figure that line IV dominates all the other lines. Also any point on line IV is superior to the points on PCQ. To see, point E on line R_fMG offers higher expected return than point B on PCQ but with the same risk. Also, point G on R_fMG provides same expected return as point D on PCQ but with lower standard deviation.

Thus if borrowing and lending at risk free rate is introduced, the efficient frontier is a straight line. See Figure .3.

Figure .3



Efficient Frontier with Lending and Borrowings

The investor now has the following choices: Invest all his wealth in portfolio M, or invest some of his wealth in portfolio M and remaining in R_f (point E, lending), or borrow more funds to invest in portfolio M (point G, borrowing).

No other investment or combination of investments available is as efficient as point M. the decision to purchase M is the investment decision. The decision to buy some riskless asset (lend) or to borrow (leverage the portfolio) is financing decision. This condition gives rise to what is called as the *separation theorem.* The theorem implies that all the investors, conservative or aggressive, should hold the same mix of stocks from the efficient set. They should use borrowing or lending to attain their preferred risk class.

5. Capital Asset Pricing Model

Capital Asset Pricing Model was developed by William Sharpe (1964), John Lintner (1965) and Jan Mossin (1966). Their work was based on the foundations of Modern Portfolio Management laid by Harry Markowitz in 1952. CAPM predicts the relationship between the risk of an asset

and its expected return. It helps in the identification of different securities (individual asset or portfolio) as overpriced or underpriced by assigning values (not exact) to these securities. This would help in selection of securities to be included in the portfolio by the risk-averse investors. CAPM talks about the ex-ante returns (returns expected in future) from an asset. This return shall compensate the investors for the following two:

- Time-premium
- Risk-premium

Time-premium: It is the compensation or reward for waiting as the current consumption of money value is sacrificed for the present or proposed to the future. It is the compensation for the loss in value of the money. This compensation should be at least equal to the returns on risk free assets as this is the minimum expected return that one can have from his investment. The risk free assets are however, subject to interest rate risk and inflation risk. They are free from default risk.
Risk- premium: It is the reward for assuming risk by the risk averse investors. His desire for return increases with the increased quantum of risk assumed by him. The risk premium is nothing but the compensation over and above the risk involved in riskless assets. The risk premium increases with the increase in risk. In other words, higher the risk higher the return; lower the risk lower the return.

5.1. Assumptions of Capital Asset Pricing Model

CAPM is based on the following assumptions:

- The investors are risk averse i.e. they attempt to avoid risk.
- Investors seek to maximize the expected utility of their portfolio over a single planning period horizon.
- Each security in the market has only two attributes, on the basis of which these are selected in the portfolio. These are mean returns and variances of such returns representing the risk.
- Investors have homogeneous expectations i.e. they have identical subjective estimates of means, variances and co-variances among returns.
- The investors can borrow and lend freely at the riskless rate of interest.
- The market is perfect, there are no taxes, there are no transaction costs, securities are completely divisible, and markets are competitive.
- The quantity of risky securities in the market is given.

The model determines the prices of securities in such a manner that the risk premium or excess returns are proportional to the systematic risk, measured through beta. It takes into account the assets' sensitivity to the non-diversifiable risk, as well as expected return on theoretical risk free asset for deriving the theoretical required return (i.e. discount rate) for an asset in the market.
Mathematically, CAPM is expressed as:

$$E(R_i) = R_f + \beta_i\{E(R_M) - R_f\}$$

.....Equation.5

Where, Beta (β) is the measure of assets sensitivity to movements in the overall market. Beta is usually found through regression on historical data. Beta more than 1 devotes more than average risk and beta less than 1 denotes less than average risk.

$[E(R_M) - R_f]$ denotes the market premium, the historically observed excess return of market over the risk free rate.

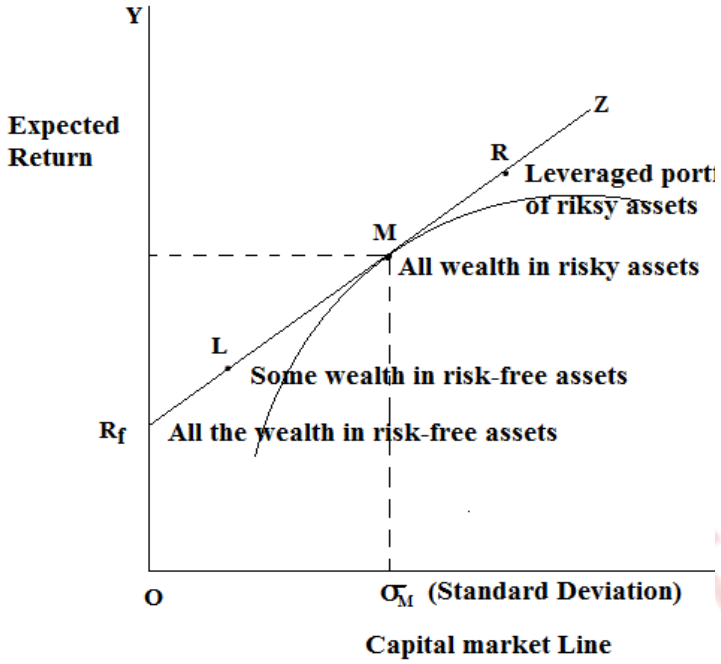
Once, the E(R), expected return is calculated using CAPM, this rate can be used for discounting the future cash flows of the asset to their preset values for establishing the correct price of the asset.

6. Capital Market Line

CAPM says that all investors will hold combinations of only two portfolios i.e. the market portfolio and the riskless security. The market portfolio is a combination of all risky assets in the market. The weight of each asset in the portfolio held by the investor will be proportional to weight of the assets in the market value. In other words, each asset will be held in the proportion of its market value to the market value of all risky assets.

CML represents this combination of risk free asset and the market portfolio. It is a straight tangent line emanating from the point of risk free asset to the set of efficient portfolios of the risky securities, i.e. the efficient frontier. The point of tangency represents the market portfolio, which is an efficient portfolio comprising all risky securities. All efficient portfolios will lie on the CML including the market portfolio. Portfolios having returns that are perfectly positively correlated with the market portfolios are referred to as efficient portfolios. All points on the CML have superior risk return profile as compared to any point on the efficient frontier except point M, the market portfolio. All investors would choose portfolios somewhere along the CML. All inefficient portfolios lie below the CML. CML is presented graphically in Figure.4.

Figure .4



Capital market Line

CML depicts the relationship between risk (standard deviation) and return for all efficient portfolios including the market portfolio. This relationship is given mathematically by;

$$E(R_P) = R_f + \{\frac{E(R_M) - R_f}{\sigma_M}\}\ \sigma_P$$

.....Equation.6

Where, $E(R_P)$ = expected return of the portfolio
R_f = risk free rate, price of time
σ_P = standard deviation of the portfolio
σ_M = standard deviation of the market portfolio
$\{\frac{E(R_M) - R_f}{\sigma_M}\}$ = slope of the CML
This can also be thought of as the extra return that can be gained by increasing the level of risk on an efficient portfolio by one unit. Thus the expected return of an efficient portfolio is;

Expected Return = (Price of Time) + (Price of Risk) * (Amount of Risk)

7. Security market Line

CAPM explains the relationship between expected return and risk. CAPM believes that in an efficient market, it is the systematic risk that is much more vital. Since the systematic risk cannot be eliminated or reduced through diversification, it demands proper management. On the other hand, the non-systematic risk can be reduced to zero through diversification; it is thus irrelevant

for investors. Hence, it is the systematic risk for which investors are rewarded or compensated. Systematic risk is measured through beta. Beta is sensitivity of security's return to the movement in the general market index or the market portfolio.

CAPM describes that expected return of a security is linearly related to its risk, measured through beta. This means higher the beta, higher the expected return and vice-versa. The basic theme of CAPM is depicted through a straight line called the Security market Line.

The individual securities and inefficient portfolios lie below the CML and are found all through the feasible region. There exists no linear relationship between their expected return and standard deviation (variances). However, there exists relationship between their expected returns and their co-variances with market portfolio. This relationship is graphically presented in the form of a straight line- security market line.

Mathematically,

$$E(R_i) = R_f + \{\frac{E(R_M) - R_f}{\sigma_M^2}\} C_{iM}$$

.....Equation.7

Where, $E(R_i)$ = expected return on security i

R_f = risk free rate

$E(R_M)$ = expected return on market portfolio

σ_M^2 = variance of return on market portfolio

C_{iM} = covariance of return between security i and market portfolio.

In other words the SML is;

Expected Return on security i = Risk free rate + (Price per unit of risk) Risk

Where, $\{\frac{E(R_M) - R_f}{\sigma_M^2}\}$ = price per unit of risk

C_{iM} = measure of risk

Since, $\beta_i = \frac{Cov_{iM}}{\sigma_M^2}$ or $\frac{\sigma_{iM}}{\sigma_M^2}$

We can express the SML as given below,

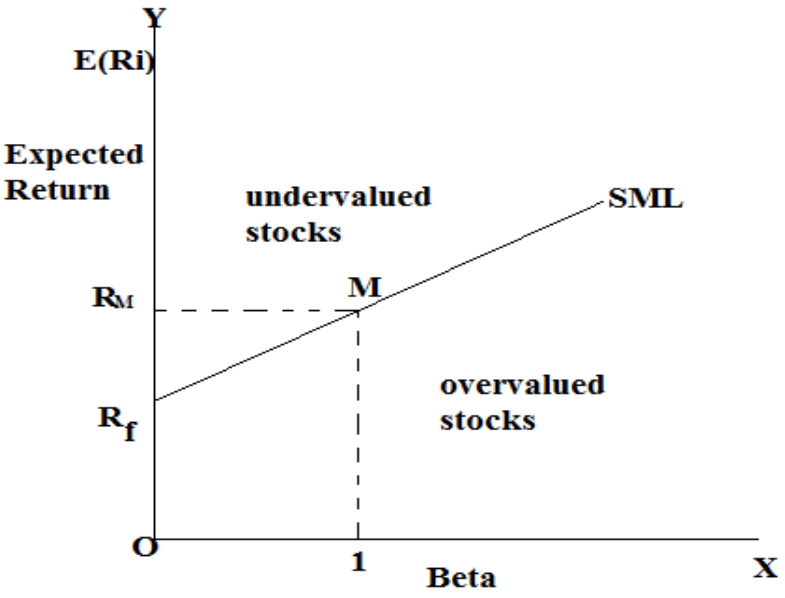
$$E(R_i) = R_f + \{E(R_M) - R_f\} \beta_i$$

.....Equation.8

Expected Return on Security i = Risk Free Return + (Market Risk Premium) * Beta of security i

Thus, SML is a straight line representing the CAPM. We determine the SML by connecting the risk- free rate (intercept of the line) to the market portfolio M (with beta= 1 and return of R_M). This is shown in Figure.5

Figure.5



Security Market Line

In Figure 36.5, the upward sloping straight line represents the Security Market Line. SML forms the efficient frontier for the identification of undervalued and overvalued securities. The securities lying above the SML are considered to be undervalued whereas those lying below the SML are considered to be overvalued. All the correctly priced securities lie exactly on the SML. The difference between the actual expected return and the fair return as per SML is called the security's alpha, denoted as α

8. Summary

- The capital market theory develops a model for determining the price of risky asset in the capital market.
- The Capital Market Theory assumes the existence of risk free asset, risk free lending and borrowing among the investors.
- With the introduction of the risk free assets, the Markowitz's efficient frontier undergoes a change. The resultant efficient frontier is a straight line.
- All investors hold only the market portfolio and the risk free securities.
- The Capital Asset Pricing Model predicts the relationship between the risk of an asset and its expected return.
- The model determines the prices of securities in such a manner that risk premium are proportional to systematic risk measured through beta. It takes into account asset's sensitivity to non-diversifiable risk as well as expected return on theoretical risk free rate. Mathematically:

$$E(R_i) = R_f + \beta_i\{E(R_M) - R_f\}$$

- Capital Market Line is a straight line emanating from the point of risk free asset to the set of efficient portfolios of risky securities, i.e. the efficient frontier.
- All efficient portfolios lie on the CML including the market portfolio.
- CML depicts the relationship between risk (standard deviation) and return for all efficient portfolios including the market portfolio. Mathematically:

$$E(R_P) = R_f + \{\frac{E(R_M) - R_f}{\sigma_M}\}\sigma_P$$

- The basic theme of CAPM is depicted through a straight line called the security market line.
- The security market line shows the linear relationship between expected returns and the betas of the securities.
- The SML is an upward sloping straight line passing through the market portfolio with an intercept at the risk free return securities. Mathematically:

$$E(R_i) = R_f + \{\frac{E(R_M) - R_f}{\sigma_M^2}\}C_{iM}$$

- The upward slope of the SML denotes that greater expected returns at company higher levels of betas.
- Return expected from an investment or portfolio is a combination of risk free asset plus market risk premium sensitive to the security.

$$E(R_i) = R_f + \{E(R_M) - R_f\}\beta_i$$